The Good, the Bad and the Ugly:
Agent Behavior and Efficiency in Open and Closed Organizations

Duncan M. Holthausen

Theofanis Tsoulouhas

Department of Economics

North Carolina State University

Raleigh, NC 27695-8110

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Abstract: This paper develops a novel model of agent behavior in organizations in order to compare the efficiency of "open" versus "closed" organizations. Closed organizations "screen" potential agents before admitting them while open organizations do not. Both have the option to "sort" (audit) individual behavior after observing aggregate outcome. Each agent is intrinsically "good," "bad" or "ugly," but can behave as any of the three types. Screening allows the organization to deny entry to the worst agent types, while sorting allows the organization to penalize opportunistic misbehavior by the agents. We show that both organizations may sort in equilibrium. When the sorting cost per agent is constant or exhibits economies of scale, surprisingly, both organizations sort the same number of agents, which leads agents of the same type to behave uniformly across organizations. However, agent behavior across intrinsic types may or may not be uniform. Interestingly, there is no equilibrium in which all agent types behave as good. When all agent types behave as bad or all behave as ugly, and an equilibrium exists in both organizations, closed organizations are less efficient from the organization's perspective than open ones. When all agent types behave as ugly, closed organizations are socially inefficient as well. If agent behavior is a mixture of types, then closed organizations can be efficient because they screen out some of the worst agent types in advance. When organizations can precommit to a sorting frequency, more equilibria exist; for instance, all agent types may behave as good.

1. Introduction

Why are some organizations or societies more open than other, even seemingly similar, organizations or societies? For example, firms in the same industry, equally developed countries or institutions engaged in the same activities often differ in how selective they are when admitting agents. In addition to this initial screening, organizations frequently monitor, audit or sort their agents *ex post*. Why do some organizations sort more intensively than others, and is there a link between the degree of screening and the intensity of sorting? How does the extent of screening and sorting affect the behavioral choices of agents and the ensuing efficiency of organizations?

We develop a novel framework in this paper to address these important questions and shed light on the actual practice of organizations. We provide an explanation based on the characteristics of the agent populations, the expected equilibrium behavior of agents, the costs of screening and sorting, and the penalty structure set by the institutional environment. The model is general enough to allow the organization to be a variety of institutions. The organization could be a firm and the agents potential employees, or a country and the agents potential immigrants, or a governmental agency enforcing laws, or a school or licensing authority dealing with applicants. For example, big city law and consulting firms are reputedly more lenient in hiring than small city firms but fewer hires make partner. Large Japanese corporations generally provide long-term employment to their employees, but in doing so they are very selective when they hire people. Some companies give tests to all applicants, for example Microsoft reputedly tests the intelligence and creativity of applicants regardless of credentials, other firms such as Home Depot require all potential employees to take drug tests, and banks routinely obtain credit reports for all job applicants. Some occupations require extensive licensure testing. Some universities admit most students who apply but flunk out a large percentage, while others are very selective but graduate most students who matriculate. It is more difficult to get into a respectable Japanese university than a comparable U.S. university, but reputedly easier to graduate. Several European countries test drivers more thoroughly before granting drivers' licenses than in the United States, but monitoring of good driving behavior (for instance, speeding) is less thorough. In many European countries government job applicants are required to present more documents (for instance criminal records) and undergo greater testing than those in the United States, but there may be less scrutiny once the applicants are hired.

Sorting can take different forms. Organizations may monitor agents while the agents are executing tasks or audit agents after they have taken actions. Examples include observation of worker performance by supervisors, periodic performance evaluations, enforcement of traffic laws, tenure decisions at universities, admission to partnership in law and accounting firms, and school testing. According to Cross (2001), many companies routinely fire 5-10% of their least productive workers over the course of a year. Companies use relative performance evaluation and compare employees to averages or to other employees (as in a tournament) to identify laggards and weed out these weak links. In some companies or institutions sorting takes the form of up-or-out contracts. While some companies reform, retrain or reassign the bottom ranked workers, other companies simply fire these employees immediately after they are identified. For instance, in the last few years, GE has instituted a program it calls "Organizational Vitality" in which bottom ranked workers are reformed successfully or they are ousted. Cisco has a plan where the bottom ranked 5% employees are put on a "Performance Improvement Plan." Employees who fail to achieve prespecified milestones are simply "PIPped." On the other hand, Siebel routinely turns over the lowest performing employees without spending resources to revitalize them. Tenure denial at academic institutions (in rates that differ among institutions) is another example.

We develop the model of agent behavior in organizations in section 2. Each agent is intrinsically "good," "bad" or "ugly," but can behave as any of the three types in equilibrium, as shown in sections 3 and 4. Organizations can be "open" or "closed." Closed organizations "screen" potential agents before admitting them while open organizations do not.. Both types of organization have the option to audit or "sort" individual behavior after observing the aggregate outcome obtained by the agents, and impose disciplinary penalties. The initial screening is designed to weed out those agents whose performance is likely to be unacceptable. In contrast, *ex post* sorting aims at isolating agents whose actual behavior is unacceptable, and one might expect that more thorough screening would reduce the scope for sorting. We show that extensive initial screening does not eliminate the scope for *ex post* sorting, because limited sorting invites opportunistic misbehavior by agents; and in equilibrium either both organizations engage in *ex post* sorting or neither one does. Surprisingly, when the sorting cost per agent is constant or exhibits economies of scale, both organizations sort the same number of agents in equilibrium, which leads agents of the same type to behave uniformly across organizations. However, agent behavior across intrinsic types may or may not be uniform. Interestingly, there is no equilibrium in which all agent types behave as good, provided that organizations do not precommit to a sorting frequency. If all agent types behaved as good, organizations would never sort agents. Agents, expecting no sorting, would never behave as good. When all agent types behave as bad or all behave as ugly, and an equilibrium exists in both organizations, closed organizations are less efficient from the organization's perspective than open ones, as shown in section 5. This follows because both organizations sort the same number of agents, so the screening costs incurred by the closed organization are unnecessary. When all agent types behave as ugly, closed organizations are socially inefficient as well. If agent behavior is a mixture of types, then closed organizations can be efficient because they screen out some of the worst agent types in advance.

Our analysis yields the following implications. Organizations facing sufficiently high screening costs or, interestingly, sufficiently high sorting costs per agent relative to penalties recouped for agent misbehavior, will choose the open organization type. If the organization believes that the proportion of ugly intrinsic types in the population from which agents are drawn is sufficiently high or sufficiently low, or the proportion of good intrinsic types is sufficiently high, it will choose the open organization type again. Organizations facing a sufficiently high outcome from agents who behave as good or bad relative to the outcome received from agents who behave as ugly will chose the closed organization type, provided that the sorting cost is sufficiently low. Lastly, organizations that can recoup sufficiently high penalties for bad or ugly behavior will choose the open type, provided that the sorting cost is sufficiently low.

We show in section 6 that most of the findings above extend to the case in which organizations can precommit to a sorting frequency. However, more equilibria are then possible; that is, equilibria exist in cases where no equilibria exist without precommitment. For instance, agent behavior in equilibrium can be uniformly good with precommitment. Thus, precommitment may or may not have value. Precommitment has no value when the sorting cost is less than the penalties expected to be recouped through sorting, because the organization will still find it optimal to sort the same number of agents. By contrast, precommitment has value when there are equilibria under precommitment while no equilibria exist under no-precommitment, and in the case in which the sorting cost exceeds the penalties expected to be recouped, because instead of sorting no agents, the organization may precommit to sorting some agents at a loss in order to induce better behavior.

We also extend the analysis to allow for decreasing returns to scale in sorting, or for sequential rather than simultaneous sorting. In both cases we find that the number of sorted agents can be intermediate; that is, other than all or none, which was the case with constant or increasing returns to scale and simultaneous sorting. Under decreasing returns to scale we also find that closed organizations will never sort more agents in equilibrium than open organizations, and when the organizations sort a different number of agents, closed organizations induce better agent behavior even though they sort fewer agents.

While this paper does not draw directly from existing literature, it is related to other work on auditing and agent behavior in organizations. For instance, Kahlil (1997) examines a principal-agent model in which the principal can audit the agent's compliance with a contract but cannot precommit to auditing. He showed that lack of commitment to auditing when information is asymmetric can lead to production above the level that would be obtained with full information in order to reduce the probability of agent noncompliance. Maskin and Tirole (2001) consider "accountable" democratic systems in which government officials are screened and disciplined by voters and "unaccountable" systems in which government officials are appointed and hence neither screened nor disciplined by voters. Their interest is in determining the circumstances under which each type of system works best.

2. Model

We develop a model with one organization and a finite number of agents. Agents are born with *intrinsic* types but then select *behavioral* types that need not correspond to their intrinsic types. An intrinsic type does not reflect innate ability or competence but rather a predisposition to, for example, work hard, behave legally or conform to organizational norms. The agents make endogenous decisions about their behavioral types by considering the benefits and costs of these decisions. The costs include the expenses associated with adopting various behavioral types contingent on the intrinsic types, and the penalty from being caught (sorted) weighted by its likelihood. Assume three intrinsic types: *intrinsically good, intrinsically bad and intrinsically ugly*; also assume three behavioral types: *behaviorally good, behaviorally bad and behaviorally ugly*. We consider two types of organization, *closed organization* and *open organization*. A closed organization screens its agents before they are admitted into the organization more extensively than does an open organization. For simplicity, we assume a closed organization screens all agents before the agents are admitted to the organization, however, an open organization does no screening at all so all agents are allowed to join. Both a closed and an open organization sort their agents at random after the agents select behavioral types.[1]

The timing of events is as follows: First, nature selects an intrinsic type $t_i \in \{g,b,u\}$ for each agent i, where g stands for intrinsically good, b for intrinsically bad and u for intrinsically ugly, with probability $p_i(t_i) > 0$ and *anonymity*, that is $p_i(t_i) = p(t_i)$, for all i. Then agents privately learn their intrinsic types. Second, the closed organization screens agents at a fixed cost of s per agent, rejecting those who are found to be the worst type, ugly, so that only n intrinsically good and bad types are let

[1] Note that for concreteness we model sorting as taking place once after the agents choose behavioral types. The model, however, could easily be extended to include sorting while behavioral types are being adopted (i.e., monitoring of agents' activities) without changing the results qualitatively.

in.[2] Assuming that the closed organization lets good and bad types in makes the analysis more interesting than if the organization only lets in good types (in which case all agents would behave the same way in equilibrium). The open organization, by contrast, does no screening and n agents, who can be intrinsically good, bad or ugly, are let in. Third, each agent i chooses a behavioral type or action $\tau_i \in \{G,B,U\}$, where G = behaviorally good, B = behaviorally bad and U = behaviorally ugly, at an adjustment cost of $k(\tau_i|t_i)$; that is, the cost of adopting a behavioral type depends on the agent's intrinsic type. Let $\tau = (\tau_1,...,\tau_n)$ be the vector of behavioral types adopted by the agents. Each action τ_i leads deterministically to a payoff to the agent of $v(\tau_i)$ and to an outcome $x_i(\tau_i)$ for the organization, with $x_i(U) = x(U)$, $x_i(B) = x(B)$, $x_i(G) = x(G)$, $\forall i$, and $x(G) > x(B) > x(U) > 0$. Fourth, even though the organization does not observe individual outcomes, it does observe the aggregate outcome,[3] $X = \sum_{i=1}^{n} x_i(\tau_i)$. Fifth, the organization sorts R agents, and R/n is the probability (frequency) that an agent is sorted. When an agent is sorted, his true behavioral type is publicly revealed. The administrative cost to sort an agent is assumed to be the same for both types of organization and is fixed and equal to z. An agent who is found to be τ_i pays a predetermined penalty $r(\tau_i)$. We assume the penalties are predetermined because the legal system, standard industry practices, organizational norms and outside agencies such as accrediting or overseeing bodies commonly predetermine or restrict the penalties for various types of behavior. Our analysis applies to these cases. Depending on the organization, the penalty could take many forms; for example, it could be a reduction in salary or a fine for illegal behavior. For simplicity we assume that the penalty imposed on the agents is paid to the organization. Qualitatively similar results would be obtained if the payoff to the organization did not coincide with the penalty but was systematically dependent on it. For instance, if a firm determines that an employee should be reformed by taking more extensive training, the employee suffers a welfare loss while the firm enjoys an increase in productivity, both dependent on the amount of training.

We make the following assumptions:

Assumption 1a. $k(G|g) = k(B|b) = k(U|u) = 0$.

1b. $k(2) = k(G|u) = k(U|g) \geq$

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[2] For simplicity, we assume that organizations can determine through screening whether agents are intrinsically ugly, but determining precisely whether agents are intrinsically good or bad is prohibitively costly. In a more complex model we could endogenize the quality of screening by the closed organization. We conjecture that the results would not change qualitatively.

[3] Note that the model can easily be expanded to include the moral hazard case in which the organization observes individual contributions, but those are stochastically dependent on unobservable actions taken by the agents.

$$\$ \ 2k(1) = 2k(B*u) = 2k(U*b) = 2k(G*b) = 2k(B*g) > 0.$$

Assumption 1a means there is no cost to the agent in adopting a good (bad) (ugly) behavioral type when his intrinsic type is also good (bad) (ugly). We assume in 1b that the cost of choosing a behavioral type depends on whether the agent chooses a behavioral type one or two steps removed from his intrinsic type. For instance an agent who is intrinsically good moves one step if he chooses to behave as bad and moves two steps if he chooses to behave as ugly. The rational for the symmetry in the behavioral adjustment cost structure is that the direction of adjustment does not matter because these are psychic or learning costs. For example, an agent who is born a hard worker will find it as difficult to adjust to shirking as it is for a born shirker to become a hard worker. An honest agent may also find it as difficult to adopt illegal behavior as it is for a criminal to adjust to living honestly. If an agent chooses a behavioral type which is two steps removed from his intrinsic type, then we assume that the cost $k(2)$ is at least twice the cost $k(1)$ of choosing a behavioral type which is only one step removed from his intrinsic type. Hence the behavioral adjustment cost is assumed to follow a "convex" pattern.

Assumption 2. $v(B) - v(G) > v(U) - v(B) > 0, v(G) > 0.$

Assumption 2 ensures that the payoff to the agent, excluding type adjustment costs $k(\cdot)$ and penalties $r(\cdot)$, follows a "strictly concave" pattern. This assumption implies $v(U) > v(B) > v(G) > 0$. For example, the payoff from shirking is higher than that from working hard because, even though the monetary reward may be less, the cost of effort is much less than that from working hard.

Assumption 3. $v(U) - k(U*b) - v(B) > 0$ and $v(U) - k(U*g) - v(G) > 0.$

Assumption 3 means that without *ex post* sorting, and hence penalties, starting from b (or g) an agent prefers to behave as U rather than as B (or G).[4] For example, absent police enforcement, many people would engage in illegal activities. This assumption makes the analysis more interesting because it means agents have an inherent incentive for misbehavior and thus there may be scope for sorting.

Assumption 4. $r(U) \ \$ \ 2r(B)$ and $r(B) > r(G) = 0.$

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[4] Recall that $k(B*b) = k(G*g) = 0.$

Assumption 4 means that the penalty imposed on agents when found behaviorally ugly is at least twice as high as that when found bad, which in turn is higher than the penalty (normalized to zero) imposed when found good.[5] This ensures that the penalty structure is "convex."

If an agent of intrinsic type t_i chooses behavioral type τ_i, his expected payoff given his adjustment cost and, if sorted, his penalty is

$$E(\tau_i|t_i) = v(\tau_i) - k(\tau_i|t_i) - \frac{\mathcal{R}}{n} r(\tau_i). \tag{2.1}$$

The open organization's expected payoff after observing the aggregate outcome X is

$$\Pi_o(\mathcal{R}|\tau,X) = \sum_{i=1}^{n} x_i(\tau_i) + \mathcal{R}\left[- z + \sum_{\tau_i} \Phi(\tau_i|X)\, r(\tau_i)\right], \tag{2.2}$$

where $\Phi(\tau_i|X) = \varphi(\tau_i|X)/n$ is the organization's assessment of the frequency of agents behaving as τ_i, made after observing X, with $\varphi(\tau_i|X)$ being the assessment of the number of agents behaving as τ_i. The assessed frequency $\Phi(\tau_i|X)$ is given by Bayes rule

$$\Phi(\tau_i|X) = \frac{\Phi(X|\tau_i)\, \Phi(\tau_i)}{\sum_{\tau_i} \Phi(X|\tau_i)\, \Phi(\tau_i)}, \tag{2.3}$$

where,

$$\Phi(\tau_i) = \sum_{t_i} p(\tau_i|t_i)\, p(t_i). \tag{2.4}$$

Note that in pure strategies, the probability $p(\tau_i|t_i)$ that agent i chooses action τ_i given his intrinsic type t_i, which enters (2.4), equals either 1 or 0.

In a closed organization, the expected payoff after observing the aggregate outcome X is

$$\Pi_c(\mathcal{R}|\tau,X) = - ms + \sum_{i=1}^{n} x_i(\tau_i) + \mathcal{R}\left[- z + \sum_{\tau_i} \Phi(\tau_i|X)\, r(\tau_i)\right], \tag{2.5}$$

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[5] Because the penalties are predetermined, the organization can utilize only the number of agents to be sorted to induce good behavior. As the analysis below will indicate, this is not sufficient to induce good behavior by all agents, therefore we cannot include incentive compatibility constraints that ensure good behavior by all agents. Note that the analysis directly generalizes to predetermined rewards as opposed to penalties.

where m is the number of agents that had to be screened to find n good and bad agents, $\Phi(\tau_i|X)$ is determined by Bayes rule as above, and

(2.6) $$\Phi(\tau_i) = \sum_{t_i} p(\tau_i|t_i)\,\mu(t_i),$$

with the probability of agent i being of type t_i in a closed organization given by

(2.7) $$\mu(t_i) = \frac{p(t_i)}{p(g) + p(b)},\ \forall t_i \in \{g,b\}.$$[6]

The rationale for (2.7) is that the closed organization screens the agents and removes all the intrinsically ugly types u. Note that this implies

(2.8) $$\mu(t_i) > p(t_i),\ \forall t_i \in \{g,b\},$$

because agents of type u are replaced with agents of types g and b. Of course, if the closed organization draws from a different population than does the open organization (e.g., a European versus a US pool), it is not possible to compare the distributions of the intrinsic types in the two organizations (that is, neither (2.7) nor (2.8) apply in this case).

In analyzing the extensive-form game played between the organization and the agents, given the type of organization, we adopt perfect Bayesian equilibrium (PBE) as the solution concept and we focus on pure strategy equilibria. The equilibrium of the game is characterized by: (i) a choice of behavioral type τ_i by each intrinsic type t_i of agent, that is, $\tau_i(g)$, $\tau_i(b)$ and $\tau_i(u)$, with $\tau_i(t_i) \in \{G,B,U\}$; (ii) a function $\Phi(\tau_i|X) \in [0,1]$ denoting the organization's beliefs about agent i's behavioral type after observing the aggregate outcome X; (iii) a strategy $R(X)$ for the organization determining the number of agents to be sorted based on the observed value of aggregate outcome X. The agents' strategies and the organization's strategy and beliefs must satisfy the following conditions. First, an agent's choice of behavioral type given his intrinsic type is sequentially rational, that is, it maximizes his expected payoff in (2.1), given the organization's strategy and belief function. Second, given the agents' strategies, the organization's observation of aggregate outcome and its belief function, the organization

[6] Note that for ease of exposition we have abused the notation; for instance, we use the same notation for all $\Phi(\cdot)$ probabilities regardless of the type of organization, even though the probabilities depend on the type of organization.

chooses the strategy that maximizes its expected payoff shown in (2.2) for an open organization, or in (2.5) for a closed organization. Third, the organization's belief function is derived from Bayes rule according to (2.3).

The following cases are possible regarding belief probabilities about behavioral types: **(i)** If X = nx(G) or X = nx(U), the organization receives a fully informative signal about each agent's behavioral type because when X = nx(G) the organization knows that $\tau_i(t_i)$ = G œi, and when X = nx(U), $\tau_i(t_i)$ = U œi. **(ii)** If X = nx(B), either all agents behave as bad or the average output obtained by agents behaving as good, φ(G*X), and those who behave as ugly, φ(U*X), is x(B), regardless of the number who behave as bad, φ(B*X). **(iii)** If nx(U) < X < nx(G) and agents behave in equilibrium as one of two types, then the organization infers the number of agents behaving as either type exactly by solving two linear equations with two unknowns. For instance, suppose agents behave as good or ugly, then φ(G*X) + φ(U*X) = n and φ(G*X)x(G) + φ(U*X)x(U) = X. If agents behave in equilibrium as one of three types, then determining the exact number who behave as each type is obviously not possible.

3. Agent Behavior in An Open Organization

We start by defining a number of parameters that will be useful in characterizing behavior in equilibrium. First, let $RBS_{BU}(t_i)$ denote the Rate of Behavioral Substitution defined as the ratio of the marginal benefit of behaving as B rather than as G divided by the marginal benefit of behaving as U rather than as G, excluding penalties if sorted and starting from any intrinsic type t_i. Thus,

(3.1) $$RBS_{BU}(t_i) = \frac{[v(B) \,\&\, k(B{*}t_i)] \,\&\, [v(G) \,\&\, k(G{*}t_i)]}{[v(U) \,\&\, k(U{*}t_i)] \,\&\, [v(G) \,\&\, k(G{*}t_i)]}.$$

The analysis below will demonstrate the importance of this rate in our results. The rationale is that agents find it tempting to adopt bad or ugly behavior rather than good; hence, equilibrium properties depend on the rate of substitution between bad and ugly behavior.

Agent behavior will also be shown to depend on whether the sorting frequency R/n lies in various intervals defined by the two parameters defined below. Let

(3.2) $$A_1(t_i) = \frac{[v(U) \,\&\, k(U{*}t_i)] \,\&\, [v(G) \,\&\, k(G{*}t_i)]}{r(U)},$$

and

(3.3) $$A_2(t_i) = \frac{[v(U) \& k(U*t_i)] \& [v(B) \& k(B*t_i)]}{r(U) \& r(B)}.$$

$A_1(t_i)$ shows the marginal benefit to the agent of behaving as U rather than as G, divided by the marginal penalty imposed on the agent if he is sorted and found to have behaved as U rather than as G (recall that r(G) = 0 which simplifies the denominator of (3.2)). $A_2(t_i)$ shows the marginal benefit to the agent of behaving as U rather than as B, divided by the marginal penalty imposed on the agent if he is sorted and found to have behaved as U rather than as B. It can easily be shown that $RBS_{BU}(t_i)$ is related to $A_1(t_i)$ and $A_2(t_i)$. In particular, for every t_i,

(3.4) $$RBS_{BU}(t_i) = 1 - \frac{A_2(t_i)}{A_1(t_i)} \frac{[r(U) \& r(B)]}{r(U)}.$$

Condition (3.4) implies that

(3.5) $$RBS_{BU}(t_i) < (=) (>) \frac{r(B)}{r(U)} \;] \; A_1(t_i) < (=) (>) A_2(t_i),$$

which means that if the Rate of Behavioral Substitution between B and U, rather than G, is smaller (larger) than the ratio of penalties if sorted, then the marginal benefit of behaving as U rather than as G, divided by the corresponding marginal penalty if sorted, is smaller (larger) than the marginal benefit of behaving as U rather than as B, divided by the marginal penalty if sorted.

For an agent of intrinsic type t_i, denote the marginal benefit of behaving as U rather than G as

(3.6) $$D(t_i) = [v(U) - k(U*t_i)] - [v(G) - k(G*t_i)] = A_1(t_i)\, r(U).$$

Note that given Assumptions 2 and 1b, it follows that D(b) > 0, and given Assumption 1a, 1b and 2 it follows that D(u) > 0, which will be useful in Lemma 1. Lemma 1 ranks the $A_1(t_i)$ and $A_2(t_i)$ values for all intrinsic types depending on the magnitude of the Rate of Behavioral Substitution.

Lemma 1. Given Assumptions 1, 2 and 4, in an open organization, if $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U), that is,

(3.7) $$RBS_{BU}(t_i) < \frac{r(B)}{r(U)} \& \frac{2r(U)[k(2)\&k(1)] \& r(B)k(2)}{D(b)r(U)} \text{ ,œ}t_i,$$

with

$$(3.8) \qquad \frac{2r(U)[k(2)-k(1)] - r(B)k(2)}{D(b)r(U)} > 0,$$

then

$$(3.9) \qquad A_1(g) < A_1(b) < A_1(u) < A_2(g) \leq A_2(b) < A_2(u),$$

and if $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U), that is,

$$(3.10) \qquad RBS_{BU}(t_i) > \frac{r(B)}{r(U)} + \frac{2k(2)}{D(u)} \frac{[r(U) - r(B)]}{r(U)}, \forall t_i,$$

with

$$(3.11) \qquad \frac{2k(2)}{D(u)} \frac{[r(U) - r(B)]}{r(U)} > 0,$$

then

$$(3.12) \qquad A_2(g) \leq A_2(b) < A_2(u) < A_1(g) < A_1(b) < A_1(u).$$

Proof. See Appendix.

Given the significance of Lemma 1 for the remaining analysis, further discussion is warranted. We focus on the polar cases where r(B) / r(U) is larger than the RBSs for all intrinsic types, or smaller than the RBSs for all intrinsic types. Cases in between can also be analyzed, but they lead to unnecessary complexity without adding any interesting results. Condition (3.7) requires that $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U). The rate is sufficiently low when the payoff structure $v(\cdot)$ is not very "concave" so that condition (3.7) holds (i.e., the agents are not very risk-averse with respect to payoffs). This can easily be seen by fixing the denominator in $RBS_{BU}(t_i)$, in which case the condition is satisfied if the numerator is small. Condition (3.10) requires that $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U). The rate is sufficiently high when the payoff structure $v(\cdot)$ is sufficiently "concave" so that condition (3.10) holds.[7]

Lemma 2 below shows that agent behavior depends on whether the sorting frequency R/n lies in various intervals defined by the $A_1(t_i)$ and $A_2(t_i)$ parameters in Lemma 1. To characterize tie breaking cases we make the following regularity assumption.

Assumption 5. If $\tilde{\tau}$ = U and $\hat{\tau}$ 0 {G,B} or if $\tilde{\tau}$ = B and $\hat{\tau}$ = G, then $\tau_i(t_i) = \tilde{\tau}$ iff $E(\tau_i = \tilde{\tau}*t_i) > E(\tau_i = \hat{\tau}*t_i)$. If $\tilde{\tau}$ = G and $\hat{\tau}$ 0 {B,U} or if $\tilde{\tau}$ = B and $\hat{\tau}$ = U, then $\tau_i(t_i) = \tilde{\tau}$ iff $E(\tau_i = \tilde{\tau}*t_i)$ \$ $E(\tau_i = \hat{\tau}*t_i)$.

This assumption means that if an agent is indifferent between two distinct behavioral types, he will adopt the "better" behavioral type. The agent adopts a "worse" behavioral type only when his payoff is strictly larger for the worse type. For instance, the agent of some given intrinsic type will behave as U rather than as B if the payoff for behaving as U is strictly greater than the payoff for behaving as B. If the payoffs are the same, the agent will behave as B.

Lemma 2. Given Assumption 5 and Lemma 1, in an open organization, if condition (3.7) holds so that $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U), then for any 0 # R/n #1:

(3.13a) If $\frac{R}{n} < A_1(g)$, then $\tau_i(t_i)$ = U, œi, œt_i;

(3.13b) if $A_1(g)$ # $\frac{R}{n} < A_1(b)$, then $\tau_i(g)$ = G, œi, and $\tau_i(t_i)$ = U, œi, œt_i.0 {b,u};

(3.13c) if $A_1(b)$ # $\frac{R}{n} < A_1(u)$, then $\tau_i(t_i)$ = G, œi, œt_i 0 {g,b}, and $\tau_i(u)$ = U, œi;

(3.13d) if $A_1(u)$ # $\frac{R}{n}$, then $\tau_i(t_i)$ = G, œi, œt_i.

Given Assumption 5 and Lemma 1, in an open organization, if condition (3.10) holds so that $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U), then for any 0 # R/n #1:

(3.14a) If $\frac{R}{n} < A_2(g)$, then $\tau_i(t_i)$ = U, œi, œt_i;

[7] By using equation (3.4) it can be shown that condition (3.10) is satisfied iff [v(B) - v(G)] > [v(U) - v(B)] + 4k(1), which supports the intuition that the payoff structure needs to be sufficiently "concave" (i.e., that the agents are sufficiently risk-averse with respect to payoffs).

(3.14b) if $A_2(g) \# \frac{R}{n} < A_2(b)$, then $\tau_i(g) = B$, œi, and $\tau_i(t_i) = U$, œi, œt_i 0 {b,u};

(3.14c) if $A_2(b) \# \frac{R}{n} < A_2(u)$, then $\tau_i(t_i) = B$, œi, œt_i 0 {g,b}, and $\tau_i(u) = U$, œi;

(3.14d) if $A_2(u) \# \frac{R}{n} < A_1(g)$, then $\tau_i(t_i) = B$, œi, œt_i;

(3.14e) if $A_1(g) \# \frac{R}{n} < A_1(b)$, then $\tau_i(t_i) = B$, œi, œt_i 0 {b,u}, and

$$\text{if } \frac{R}{n} < A_1(g)\frac{RBS_{BU}(g)}{\frac{r(B)}{r(U)}}, \text{ then } \tau_i(g) = B, \text{ œi, but}$$

$$\text{if } \frac{R}{n} \ \$\ A_1(g)\frac{RBS_{BU}(g)}{\frac{r(B)}{r(U)}}, \text{ then } \tau_i(g) = G, \text{ œi;}$$

(3.14f) if $A_1(b) \# \frac{R}{n} < A_1(u)$, then $\tau_i(u) = B$, œi, and

$$\text{if } \frac{R}{n} < A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, t_i \text{ 0 \{g,b\}, then } \tau_i(t_i) = B, \text{ œi, œ}t_i \text{ 0 \{g,b\}, but}$$

$$\text{if } \frac{R}{n} \ \$\ A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, t_i \text{ 0 \{g,b\}, then } \tau_i(t_i) = G, \text{ œi, œ}t_i \text{ 0 \{g,b\};}$$

(3.14g) if $A_1(u) \# \frac{R}{n}$, then

$$\text{if } \frac{R}{n} < A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, \ t_i \text{ 0 \{g,b,u\}, then } \tau_i(t_i) = B, \text{ œi, œ}t_i\text{, but}$$

$$\text{if } \frac{R}{n} \ \$\ A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, \ t_i \text{ 0 \{g,b,u\}, then } \tau_i(t_i) = G, \text{ œi, œ}t_i.$$

Proof. See Appendix.

The behavior of agents can be summarized in the following tables. To simplify notation, let

$$A_3(t_i) = A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}} = \frac{[v(B) \& k(B{*}t_i)] \& [v(G) \& k(G{*}t_i)]}{r(B)}. \tag{3.15}$$

$A_3(t_i)$ reflects the marginal benefit of behaving as B rather than as G relative to the marginal cost. It is useful for the remaining analyses to note that Assumptions 1, 2 and 3 imply

$$A_3(g) < A_3(b) \# A_3(u). \tag{3.16}$$

		R/n < $A_1(g)$	R/n < $A_1(b)$	R/n < $A_1(u)$	R/n \$ $A_1(u)$
t_i	g	U	G	G	G
	b	U	U	G	G
	u	U	U	U	G

Table 1. Behavioral type chosen by agent in an open organization when $RBS_{BU}(\cdot)$ is small.

		R/n < $A_2(g)$	R/n < $A_2(b)$	R/n < $A_2(u)$	R/n < $A_1(g)$	R/n < $A_1(b)$		R/n < $A_1(u)$		R/n \$$A_1(u)$	
						R/n< $A_3(g)$	R/n\$ $A_3(g)$	R/n< $A_3(t_i)$ t_i 0 {g,b}	R/n\$ $A_3(t_i)$ t_i 0 {g,b}	R/n< $A_3(t_i)$ œ t_i	R/n\$ $A_3(t_i)$ œ t_i
t_i	g	U	B	B	B	B	G	B	G	B	G
	b	U	U	B	B	B	B	B	G	B	G
	u	U	U	U	B	B	B	B	B	B	G

Table 2. Behavioral type chosen by agent in an open organization when $RBS_{BU}(\cdot)$ is large.

Observe that for any given value of R/n, at most only two behavioral types are chosen by agents (U and G when $RBS_{BU}(\cdot)$ is small and U and B or G and B when $RBS_{BU}(\cdot)$ is large). In the small $RBS_{BU}(\cdot)$ case behavior is extreme, specifically agents adopt only ugly or good behavior. The intuition for this result is that when $RBS_{BU}(\cdot)$ is small, agents are not very risk averse with respect to payoffs, as argued in the discussion of Lemma 1. Therefore, they are willing to accept the risk of behaving as ugly provided that the probability of being caught ($\mathbb{R}/n$) is low. However, when the chance of being caught is high, they adopt good behavior. Interestingly, they never behave as bad when the chance of getting caught is high, because a relatively small $RBS_{BU}(\cdot)$ is indicative of a small marginal benefit of behaving as bad rather than as good.

Behavior in the large $RBS_{BU}(\cdot)$ case is richer. In this case, the marginal benefit of behaving as bad rather than as good is more substantial and agents are more risk averse than in the small $RBS_{BU}(\cdot)$ case. Therefore the agents behave as ugly only if the probability of being caught is very low. If the probability is intermediate in value, agents tend to behave as bad to reduce the penalty they face if they are sorted. If the probability of being caught is high, agents behave as bad or good. They behave as bad if the probability of being caught is not very high, given that the marginal benefit of behaving as bad rather than as good is large. By contrast, if the probability of being caught is very high, they behave as good.

Given the findings in Lemma 2, Proposition 3 characterizes the equilibrium of the extensive game played between the open organization and the agents. The proposition highlights the importance of the sorting cost z relative to the expected penalty recouped from the sorted agents. It also emphasizes the significance of the $A_1(t_i)$, $A_2(t_i)$ and $A_3(t_i)$ values which, again, reflect the marginal benefit of behaving as U rather than as G, U rather than as B, and B rather than as G, respectively, relative to the penalty imposed if sorted. Proposition 3 is long because of the variety of possible cases. We give the underlying intuition immediately following the proposition, so the reader may skip over the details without losing the essence of the results.

Proposition 3. Let $E(r) = \sum_{\tau_i} \Phi(\tau_i|X)\, r(\tau_i(t_i))$ be the expected penalty from sorting agent i, and let $\Lambda(\tau_i)$ be the true frequency of agents behaving as τ_i. Given Assumption 3 and Lemma 2, in an open organization, if condition (3.7) holds so that $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U), then in the PBE of the extensive-form game the following cases are possible:

Case (i): $z \geq E(r)$

Agent behavior:	$\tau_i(t_i) = U, \forall i, \forall t_i$
Outcome observed:	$X = nx(U)$
Assessments:	$\Phi(U|nx(U)) = \Lambda(U) = 1$
Expected penalty:	$E(r) = r(U), \forall i$

Number of agents sorted: $\mathcal{R}(nx(U)) = 0$

Case (ii): z < E(r) and $A_1(g) > 1$

Agent behavior: $\tau_i(t_i)$ = U, œi, œt_i

Outcome observed: X = nx(U)

Assessments: $\Phi(U*nx(U)) = \Lambda(U) = 1$

Expected penalty: E(r) = r(U), œi

Number of agents sorted: $\mathcal{R}(nx(U)) = n$

Case (iii): z < E(r), $A_1(g)$ # 1 and $A_1(b) > 1$

Agent behavior: $\tau_i(g)$ = G, œi, and $\tau_i(t_i)$ = U, œi, œt_i 0 {b,u}

Outcome observed: $nx(U) < X = \varphi(G*X)x(G) + \varphi(U*X)x(U) < nx(G)$

Assessments: $\Phi(G*X) = \Lambda(G)$, $\Phi(U*X) = \Lambda(U)$

Expected penalty: $E(r) = \Phi(U*X)\, r(U)$, œi

Number of agents sorted: $\mathcal{R}(X) = n$

Case (iv): z < E(r), $A_1(b)$ # 1 and $A_1(u) > 1$

Agent behavior: $\tau_i(t_i)$ = G, œi, œt_i 0 {g,b} and $\tau_i(u)$ = U, œi

Outcome observed: $nx(U) < X = \varphi(G*X)x(G) + \varphi(U*X)x(U) < nx(G)$

Assessments: $\Phi(G*X) = \Lambda(G)$, $\Phi(U*X) = \Lambda(U)$

Expected penalty: $E(r) = \Phi(U*X)\, r(U)$, œi

Number of agents sorted: $\mathcal{R}(X) = n$

Given Assumption 3 and Lemma 2, in an open organization, if condition (3.10) holds, so that $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U), then in the PBE of the extensive-form game the following cases are possible:

Case (i): z $ E(r)

Agent behavior: $\tau_i(t_i)$ = U, œi, œt_i

Outcome observed: X = nx(U)

Assessments: $\Phi(U*nx(U)) = \Lambda(U) = 1$

Expected penalty: E(r) = r(U), œi

Number of agents sorted: $\mathcal{R}(nx(U)) = 0$

Case (ii): z < E(r) and $A_2(g) > 1$

Agent behavior: $\tau_i(t_i)$ = U, œi, œt_i

Outcome observed: X = nx(U)

Assessments: $\Phi(U*nx(U)) = \Lambda(U) =1$

Expected penalty: E(r) = r(U), œi

Number of agents sorted: R(nx(U)) = n

Case (iii): z < E(r), A_2(g) # 1 and A_2(b) > 1

Agent behavior: τ_i(g) = B, œi, and $\tau_i(t_i)$ = U, œi, œt_i 0 {b,u}

Outcome observed: nx(U) < X = φ(B*X)x(B) + φ(U*X)x(U) < nx(B)

Assessments: Φ(B*X) = Λ(B), Φ(U*X) = Λ(U)

Expected penalty: E(r) = Φ(B*X) r(B) + Φ(U*X) r(U), œi

Number of agents sorted: R(X) = n

Case (iv): z < E(r), A_2(b) # 1 and A_2(u) >1

Agent behavior: $\tau_i(t_i)$ = B, œi, œt_i 0 {g,b} and τ_i(u) = U, œi

Outcome observed: nx(U) < X = φ(B*X)x(B) + φ(U*X)x(U) < nx(B)

Assessments: Φ(B*X) = Λ(B), Φ(U*X) = Λ(U)

Expected penalty: E(r) = Φ(B*X) r(B) + Φ(U*X) r(U), œi

Number of agents sorted: R(X) = n

Case (v): z < E(r), A_2(u) # 1 and A_1(g) >1

Agent behavior: $\tau_i(t_i)$ = B, œi, œt_i

Outcome observed: X = nx(B)

Assessments: Φ(B*X) = Λ(B) = 1

Expected penalty: E(r) = r(B), œi

Number of agents sorted: R(nx(B)) = n

Case (vi): z < E(r), A_1(g) # 1, A_3(g) >1,[8] and A_1(b) >1

Agent behavior: $\tau_i(t_i)$ = B, œi, œt_i

Outcome observed: X = nx(B)

Assessments: Φ(B*X) = Λ(B) = 1

Expected penalty: E(r) = r(B), œi

Number of agents sorted: R(nx(B)) = n

Case (vii): z < E(r), A_3(g) # 1 and A_1(b) >1

Agent behavior: τ_i(g) = G, œi, and $\tau_i(t_i)$ = B, œi, œt_i 0 {b,u}

Outcome observed: nx(B) < X = φ(G*X)x(G) + φ(B*X)x(B) < nx(G)

Assessments: Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)

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[8] Recall that $A_3(t_i)$ was defined in (3.15).

Expected penalty: E(r) = Φ(B*X) r(B), œi

Number of agents sorted: R(X) = n

Case (viii): z < E(r), A_1(b) # 1, A_3(g) >1, and A_1(u) >1

Agent behavior: $\tau_i(t_i)$ = B, œi, œt_i

Outcome observed: X = nx(B)

Assessments: Φ(B*X) = Λ(B) = 1

Expected penalty: E(r) = r(B), œi

Number of agents sorted: R(nx(B)) = n

Case (ix): z < E(r), A_1(b) # 1, A_3(g) # 1, A_3(b) > 1 and A_1(u) >1

Agent behavior: τ_i(g) = G, œi, and $\tau_i(t_i)$ = B, œi, œt_i 0 {b,u}

Outcome observed: nx(B) < X = φ(G*X)x(G) + φ(B*X)x(B) < nx(G)

Assessments: Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)

Expected penalty: E(r) = Φ(B*X) r(B), œi

Number of agents sorted: R(X) = n

Case (x): z < E(r), A_3(b) # 1 and A_1(u) >1

Agent behavior: $\tau_i(t_i)$ = G, œi, œt_i 0 {g,b}, and τ_i(u) = B, œi

Outcome observed: nx(B) < X = φ(G*X)x(G) + φ(B*X)x(B) < nx(G)

Assessments: Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)

Expected penalty: E(r) = Φ(B*X) r(B), œi

Number of agents sorted: R(X) = n

Case (xi): z < E(r), A_1(u) # 1, A_3(g) >1, A_3(b) > 1 and A_3(u) >1

Agent behavior: $\tau_i(t_i)$ = B, œi, œt_i

Outcome observed: X = nx(B)

Assessments: Φ(B*X) = Λ(B) = 1

Expected penalty: E(r) = r(B), œi

Number of agents sorted: R(nx(B)) = n

Case (xii): z < E(r), A_1(u) # 1, A_3(g) # 1, A_3(b) > 1 and A_3(u) >1

Agent behavior: τ_i(g) = G, œi, and $\tau_i(t_i)$ = B, œi, œt_i 0 {b,u}

Outcome observed: nx(B) < X = φ(G*X)x(G) + φ(B*X)x(B) <nx(G)

Assessments: Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)

Expected penalty: E(r) = Φ(B*X) r(B), œi

Number of agents sorted: R(X) = n

Case (xiii): $z < E(r)$, $A_1(u)$ #1, $A_3(g)$ # 1, $A_3(b)$ # 1 and $A_3(u) > 1$

Agent behavior:	$\tau_i(t_i) = G$, œi, œt_i 0 {g,b}, and $\tau_i(u) = B$, œi
Outcome observed:	$nx(B) < X = \varphi(G*X)x(G) + \varphi(B*X)x(B) < nx(G)$
Assessments:	$\Phi(G*X) = \Lambda(G)$, $\Phi(B*X) = \Lambda(B)$
Expected penalty:	$E(r) = \Phi(B*X)\, r(B)$, œi
Number of agents sorted:	$R(X) = n$

Proof. See Appendix.

The rationale behind Proposition 3 is as follows. In deciding whether to sort agents or not, after observing the aggregate outcome X, the organization compares the sorting cost z to the penalties it expects to collect, E(r). Since z is a constant, the organization will sort either all agents or none depending on the relative value of z.[9] When the sorting cost exceeds the expected penalty from sorting each agent, the organization will not sort any agents, and all agents will behave as ugly. This is case (i). By contrast, when E(r) exceeds z, the organization will sort all agents, so that R/n equals 1. Then agent behavior in equilibrium depends on whether $A_1(\cdot)$ or $A_2(\cdot)$ are larger or smaller than 1. Agent behavior in equilibrium is consistent with Lemma 2 which is summarized in Tables 1 and 2. When $A_1(\cdot)$ is larger than 1, the marginal benefit of behaving as U rather than as G exceeds the marginal cost if sorted, and agents will prefer to behave as U rather than G. The opposite is true when $A_1(\cdot)$ is smaller than 1. When $A_2(\cdot)$ is larger than 1, the marginal benefit of behaving as U rather than as B exceeds the marginal cost if sorted, and agents will prefer to behave as U rather than B. The opposite is true when $A_2(\cdot)$ is smaller than 1. When $A_3(\cdot)$ is larger than 1, the marginal benefit of behaving as B rather than as G exceeds the marginal cost if sorted, and agents will prefer to behave as B rather than as G.[10]

When the Rate of Behavioral Substitution is sufficiently small, all $A_2(\cdot)$ values are greater than 1 for all intrinsic types, meaning that all agent types prefer U to B, and thus agents basically choose only between U and G, which is consistent with agent behavior as shown in Table 1. If even the good

[9] The analysis below will demonstrate that our results are quite general and apply to cases where z is not constant as well (see the extensions in section 6 below).

[10] In the small $RBS_{BU}(\cdot)$ case, $A_3(\cdot)$ larger than 1 also implies that $A_1(\cdot)$ is larger than 1, so that agents prefer both B and U to G. Clearly, some combinations of $A_1(\cdot)$, $A_2(\cdot)$ and $A_3(\cdot)$ will even lead to complete rankings of behavioral types for the agents. For instance, if $A_1(t_i)$ is larger than 1 and $A_2(t_i)$ is smaller than 1, agents of type t_i will prefer B to U to G. However, complete rankings are not necessary to characterize the equilibrium because all that is needed is a behavioral type which is preferred to any other type.

agents have strong incentives to behave as ugly rather than as good when all agents are sorted, then all agents will behave as ugly. This is case (ii) and corresponds to the first column of results in Table 1. Clearly the other cases occur when bad or ugly agent types have inherent incentives to misbehave, but good types do not. In reference to the last column of Table 1, if $A_1(u)$ # R/n = 1, all agents would behave as G if the organization sorted all agents; however, when all agents behave as G the organization will not sort any agents. Hence, there is no PBE is this case.

When the Rate of Behavioral Substitution is sufficiently large, behavior is more complex. In cases (ii) - (iv), the $A_1(\cdot)$ values are greater than 1 for all intrinsic types, meaning that all agent types prefer U to G, and thus agents basically choose only between U and B. In the remaining cases, $A_2(\cdot)$ is smaller than 1 for all intrinsic types, meaning that all agent types prefer B to U, and thus agents basically choose only between B and G. Note that if $A_1(u)$ # 1 and $A_3(t_i)$ # 1 for all t_i, then all agents would behave as G if the organization sorted all agents; however, when all agents behave as G the organization will not sort any agents. Hence, there is no PBE in this case, which corresponds to the last column in Table 2. In general, the Corollary to Proposition 3 characterizes a condition under which no PBE exists.

Corollary to Proposition 3. Let E(r) be the expected penalty from any putative equilibrium in which E(r) # z < r(U). Then the putative equilibrium is not an equilibrium.

Proof. See Appendix.

The case in which all agents behave as G satisfies the condition in the corollary because E(r) = 0. Thus, again, there is no equilibrium in which all agent types behave as G. In general, the condition E(r) < r(U) is satisfied only when some or all agent types behave as B or G. If, in addition, E(r) # z < r(U), then there is no equilibrium. The intuition is that if in the putative equilibrium E(r) # z < r(U), then the organization would not sort any agents and all agent types would behave as ugly, which contradicts the condition above because E(r) would equal r(U) in the putative equilibrium. Note that the case in which z exceeds E(r), so that the organization does no sorting and all agents behave as ugly, does not fit the condition above because z $ E(r) = r(U) in this putative equilibrium.[11]

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[11] This is equilibrium case (i) in Proposition 3.

4. Agent Behavior in a Closed Organization

Recall that a closed organization screens all agents, rejecting those who are found to be intrinsically ugly so that only n intrinsically good and bad types are let in. Therefore, the analysis of a closed organization is similar to that of an open organization, modified to eliminate u as an acceptable intrinsic type. We focus again on the two polar cases regarding the Rate of Behavioral Substitution analyzed in section 3. In the first case $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U) for all intrinsic types as in condition (3.7). In the second case $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U) for all intrinsic types as in condition (3.10). Lemma 4 below characterizes agent behavior for the two cases depending on the sorting frequency R/n in relation to the $A_1(t_i)$ and $A_2(t_i)$ values.

Lemma 4. Given Assumption 5 and Lemma 1, in a closed organization, if condition (3.7) holds so that $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U), then for any $0 \le R/n \le 1$:

(4.1a) If $\frac{R}{n} < A_1(g)$, then $\tau_i(t_i) = U, \ \forall i, \forall t_i$;

(4.1b) if $A_1(g) \le \frac{R}{n} < A_1(b)$, then $\tau_i(g) = G, \forall i$, and $\tau_i(b) = U, \ \forall i$;

(4.1c) if $A_1(b) \le \frac{R}{n}$, then $\tau_i(t_i) = G, \forall i, \forall t_i$.

Given Assumption 5 and Lemma 1, in a closed organization, if condition (3.10) holds so that $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U), then for any $0 \le R/n \le 1$:

(4.2a) If $\frac{R}{n} < A_2(g)$, then $\tau_i(t_i) = U, \ \forall i, \forall t_i$;

(4.2b) if $A_2(g) \le \frac{R}{n} < A_2(b)$, then $\tau_i(g) = B, \forall i$, and $\tau_i(b) = U, \ \forall i$;

(4.2c) if $A_2(b) \le \frac{R}{n} < A_1(g)$, then $\tau_i(t_i) = B, \forall i, \forall t_i$;

(4.2d) if $A_1(g) \le \frac{R}{n} < A_1(b)$, then $\tau_i(b) = B, \forall i$, and

$$\text{if } \frac{R}{n} < A_1(g)\frac{RBS_{BU}(g)}{\frac{r(B)}{r(U)}}, \text{ then } \tau_i(g) = B, \forall i, \text{ but}$$

$$\text{if } \frac{R}{n} \geq A_1(g)\frac{RBS_{BU}(g)}{\frac{r(B)}{r(U)}}, \text{ then } \tau_i(g) = G, \forall i;$$

(4.2e) if $A_1(b) \leq \frac{R}{n}$, then

$$\text{if } \frac{R}{n} < A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, \ t_i \in \{g,b\}, \text{ then } \tau_i(t_i) = B, \forall i, \forall t_i, \text{ but}$$

$$\text{if } \frac{R}{n} \geq A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, \ t_i \in \{g,b\}, \text{ then } \tau_i(t_i) = G, \forall i, \forall t_i.$$

Proof. The proof is analogous to the proof of Lemma 2 in the Appendix.

The intuition behind the results is similar to the intuition behind the results in Lemma 2. The only difference is that now there are no intrinsically ugly types in the organization, which reduces the number of cases to be considered. The behavior of agents can be summarized in the following tables. Similar to section 3, to simplify notation we define $A_3(t_i)$ as in (3.15).

		$R/n < A_1(g)$	$R/n < A_1(b)$	$R/n \geq A_1(b)$
t_i	g	U	G	G
	b	U	U	G

Table 3. Behavioral type chosen by agent in a closed organization when $RBS_{BU}(\cdot)$ is small.

		$R/n < A_2(g)$	$R/n < A_2(b)$	$R/n < A_1(g)$	$R/n < A_1(b)$		$R/n \geq A_1(b)$	
					$R/n < A_3(t_i)$	$R/n \geq A_3(t_i)$	$R/n < A_3(t_i)$	$R/n \geq A_3(t_i)$
t_i	g	U	B	B	B	G	B	G
	b	U	U	B	B	B	B	G

Table 4. Behavioral type chosen by agent in a closed organization when $RBS_{BU}(\cdot)$ is large.

Proposition 5 characterizes the equilibrium of the extensive game played between the closed organization and the agents. Similar to Proposition 3, Proposition 5 highlights the importance of the sorting cost z relative to the expected penalty recouped from the sorted agents, and the significance of the marginal benefit of behaving as U rather than as G or B, or B rather than as G, relative to the penalty imposed if sorted (i.e., the significance of the $A_1(\cdot)$, $A_2(\cdot)$ and $A_3(\cdot)$ values).

Proposition 5. Let $E(r) = j_{\tau_i}\ \Phi(\tau_i{*}X)\ r(\tau_i(t_i))$ be the expected penalty from sorting agent i, and let $\Lambda(\tau_i)$ be the true frequency of agents behaving as τ_i. Given Assumption 3 and Lemma 4, in a closed organization, if condition (3.7) holds so that $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U), then in the PBE of the extensive-form game the following cases are possible:

Case (i): z $ E(r)

Agent behavior:	$\tau_i(t_i)$ = U, œi, œt_i
Outcome observed:	X = nx(U)
Assessments:	Φ(U*nx(U)) = Λ(U) = 1
Expected penalty:	E(r) = r(U), œi
Number of agents sorted:	R(nx(U)) = 0

Case (ii): z < E(r) and A_1(g) > 1

Agent behavior:	$\tau_i(t_i)$ = U, œi, œt_i
Outcome observed:	X = nx(U)
Assessments:	Φ(U*nx(U)) = Λ(U) = 1
Expected penalty:	E(r) = r(U), œi
Number of agents sorted:	R(nx(U)) = n

Case (iii): z < E(r), A_1(g) # 1 and A_1(b) > 1

Agent behavior:	τ_i(g) = G, œi, and τ_i(b) = U, œi
Outcome observed:	nx(U) < X = φ(G*X)x(G) + φ(U*X)x(U) <nx(G)
Assessments:	Φ(G*X) = Λ(G), Φ(U*X) = Λ(U)
Expected penalty:	E(r) = Φ(U*X) r(U), œi
Number of agents sorted:	R(X) = n

Given Assumption 3 and Lemma 4, in a closed organization, if condition (3.10) holds, so that $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U), then in the PBE of the extensive-form game the following cases are possible:

Case (i): z $ E(r)

Agent behavior: $\tau_i(t_i) = U$, œi, œt_i

Outcome observed: $X = nx(U)$

Assessments: $\Phi(U^*nx(U)) = \Lambda(U) = 1$

Expected penalty: $E(r) = r(U)$, œi

Number of agents sorted: $R(nx(U)) = 0$

Case (ii): $z < E(r)$ and $A_2(g) > 1$

Agent behavior: $\tau_i(t_i) = U$, œi, œt_i

Outcome observed: $X = nx(U)$

Assessments: $\Phi(U^*nx(U)) = \Lambda(U) = 1$

Expected penalty: $E(r) = r(U)$, œi

Number of agents sorted: $R(nx(U)) = n$

Case (iii): $z < E(r)$, $A_2(g)$ # 1 and $A_2(b) > 1$

Agent behavior: $\tau_i(g) = B$, œi, and $\tau_i(b) = U$, œi

Outcome observed: $nx(U) < X = \varphi(B^*X)x(B) + \varphi(U^*X)x(U) < nx(B)$

Assessments: $\Phi(B^*X) = \Lambda(B)$, $\Phi(U^*X) = \Lambda(U)$

Expected penalty: $E(r) = \Phi(B^*X)\, r(B) + \Phi(U^*X)\, r(U)$, œi

Number of agents sorted: $R(X) = n$

Case (iv): $z < E(r)$, $A_2(b)$ # 1 and $A_1(g) >1$

Agent behavior: $\tau_i(t_i) = B$, œi, œt_i

Outcome observed: $X = nx(B)$

Assessments: $\Phi(B^*X) = \Lambda(B) = 1$

Expected penalty: $E(r) = r(B)$, œi

Number of agents sorted: $R(nx(B)) = n$

Case (v): $z < E(r)$, $A_1(g)$ # 1, $A_3(g) >1$ and $A_1(b) >1$

Agent behavior: $\tau_i(t_i) = B$, œi, œt_i

Outcome observed: $X = nx(B)$

Assessments: $\Phi(B^*X) = \Lambda(B) = 1$

Expected penalty: $E(r) = r(B)$, œi

Number of agents sorted: $R(nx(B)) = n$

Case (vi): $z < E(r)$, $A_3(g)$ # 1 and $A_1(b) >1$

Agent behavior: $\tau_i(g) = G$, œi, and $\tau_i(b) = B$, œi

Outcome observed: $nx(B) < X = \varphi(G^*X)x(G) + \varphi(B^*X)x(B) < nx(G)$

Assessments:	Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)
Expected penalty:	E(r) = Φ(B*X) r(B), œi
Number of agents sorted:	R(X) = n

Case (vii): z < E(r), A_1(b) # 1, A_3(g) >1

Agent behavior:	$\tau_i(t_i)$ = B, œi, œt_i
Outcome observed:	X = nx(B)
Assessments:	Φ(B*X) = Λ(B) = 1
Expected penalty:	E(r) = r(B), œi
Number of agents sorted:	R(nx(B)) = n

Case (viii): z < E(r), A_1(b) # 1, A_3(g) # 1 and A_3(b) > 1

Agent behavior:	τ_i(g) = G, œi, and τ_i(b) = B, œi
Outcome observed:	nx(B) < X = φ(G*X)x(G) + φ(B*X)x(B) < nx(G)
Assessments:	Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)
Expected penalty:	E(r) = Φ(B*X) r(B), œi
Number of agents sorted:	R(X) = n

Proof. The proof is similar to the proof of Proposition 3 in the Appendix.

Similar to the open organization, there are certain cases in which no PBE exists for the closed organization, as the corollary below demonstrates.

Corollary to Proposition 5. Let E(r) be the expected penalty from any putative equilibrium in which E(r) # z < r(U). Then the putative equilibrium is not an equilibrium.

Proof. The proof is identical to the proof of the Corollary to Proposition 3.

5. Open versus Closed Organization

In this section we focus on the efficiency of open versus closed organizations from the organization's perspective, which has implications for the choice of organizational type faced by a would-be organization drawing agents from a given pool. At the end of the section we discuss implications for social efficiency. To analyze organizational efficiency, we use the findings obtained in sections 3 and 4 which are summarized below. We start with cases in which the sorting cost per agent either exceeds the penalty expected to be received from any sorted agent in both open and closed organizations, or does not exceed the expected penalty in both organizations.

In equilibrium, both types of organization sort either no agents or all agents. When the sorting cost per agent, z, exceeds the penalty expected to be received from any sorted agent, E(r), all agents behave as U in both types of organization, regardless of the Rate of Behavioral Substitution, and the organization sorts no agents (R = 0). Thus, our first result regarding efficiency is:

Result 1. In equilibrium, when z $ r(U) in both the open and closed organization, a closed organization is unambiguously inefficient regardless of the Rate of Behavioral Substitution. This is so because both organization types sort no agents and all agents behave as ugly, but the closed organization expends resources screening agents before they are admitted to the organization.

Suppose now that z < E(r), then the organization sorts all agents (R = n, so that R/n = 1). When the Rate of Behavioral Substitution is sufficiently small, agent behavior in equilibrium is summarized in the following tables.

		$1 < A_1(g)$ (a)	$1 < A_1(b)$ (b)	$1 < A_1(u)$ (c)
t_i	g	U	G	G
	b	U	U	G
	u	U	U	U

Table 5. Equilibrium behavior chosen by agent in an open organization when $RBS_{BU}(\cdot)$ is small.

		$1 < A_1(g)$ (a)	$1 < A_1(b)$ (b)
t_i	g	U	G
	b	U	U

Table 6. Equilibrium behavior chosen by agent in a closed organization when $RBS_{BU}(\cdot)$ is small.

And when the Rate of Behavioral Substitution is sufficiently large, agent behavior in equilibrium is summarized in the following tables.

						$1 < A_1(b)$		$1 < A_1(u)$		$1 \geq A_1(u)$
		$1 < A_2(g)$	$1 < A_2(b)$	$1 < A_2(u)$	$1 < A_1(g)$	$1 < A_3(g)$	$1 \geq A_3(g)$	$1 < A_3(t_i)$ $t_i \in \{g,b\}$	$1 \geq A_3(t_i)$ $t_i \in \{g,b\}$	$1 < A_3(t_i)$ $\forall t_i$
		(a)	(b)	(c1)	(c2)	(d1)	(d2)	(e1)	(e2)	(e3)
t_i	g	U	B	B	B	B	G	B	G	B
	b	U	U	B	B	B	B	B	G	B
	u	U	U	U	B	B	B	B	B	B

Table 7. Equilibrium behavior chosen by agent in an open organization when $RBS_{BU}(\cdot)$ is large.

					$1 < A_1(b)$		$1 \geq A_1(b)$
		$1 < A_2(g)$	$1 < A_2(b)$	$1 < A_1(g)$	$1 < A_3(g)$	$1 \geq A_3(g)$	$1 < A_3(t_i)$ $t_i \in \{g,b\}$
		(a)	(b)	(c)	(d1)	(d2)	(e)
t_i	g	U	B	B	B	G	B
	b	U	U	B	B	B	B

Table 8. Equilibrium behavior chosen by agent in a closed organization when $RBS_{BU}(\cdot)$ is large.

Suppose the Rate of Behavioral Substitution is sufficiently small. Tables 5 and 6 demonstrate that if $1 < A_1(g)$ (which is case (a) in the tables), that is, if the marginal benefit to agents of intrinsic type g (and hence of any worse type) of behaving as U rather than as G exceeds the marginal cost if sorted then all agent types behave as U regardless of the organization type.

If $A_1(g) \leq 1 < A_1(b)$ (which is case (b) in Tables 5 and 6), that is, if the marginal benefit to agents of type b (and hence u) of behaving as U rather than as G exceeds the marginal cost if sorted, and if the marginal benefit to agents of type g of behaving as U rather than as G does not exceed the marginal cost if sorted, then agent types b (and agent types u, if present) behave as U, while agent types g behave as G in both organization types. Conditions (2.8) (which state that the probabilities of being intrinsically good or bad in a closed organization are larger than the corresponding probabilities in an open organization) imply that more agents behave as good in a closed organization than in an open organization. Therefore, the closed organization is expected to make a larger outcome and to recoup less in penalties from the sorted agents.

If $A_1(b) \# 1 < A_1(u)$ (which is case (c) in Table 5), that is, if the marginal benefit to agents of type u for behaving as U rather than as G exceeds the marginal cost if sorted, and if the marginal benefit to agents of type b for behaving as U rather than as G does not exceed the marginal cost if sorted, then in equilibrium in an open organization u types behave as U, while g and b types behave as G. Note that there is no equilibrium in a closed organization when $A_1(b) \# 1$ regardless of z because, when there are no u types present, if all agents are sorted they behave as G, but the organization would not sort any agents if they behaved in this manner. For similar reasons, if $A_1(u) \# 1$, no equilibrium exists in either organization type regardless of z.

To conclude the case when the Rate of Behavioral Substitution is sufficiently small, we present our second set of equilibrium results comparing the two organization types:

Result 2. In equilibrium, when $z < r(U)$, $1 < A_1(g)$ and the Rate of Behavioral Substitution is sufficiently small, a closed organization is unambiguously inefficient. This is so because even though both organization types obtain the same outcome and recoup the same penalties, the closed organization expends resources screening agents before they are admitted to the organization.

Result 3. In equilibrium, when $z < E(r)$, $A_1(g) \# 1 < A_1(b)$ and the Rate of Behavioral Substitution is sufficiently small, a closed organization may or may not be inefficient. This is so because even though the closed organization expects a larger outcome than the open organization, it expends resources screening agents before they are admitted to the organization and also expects to recoup less in penalties.

Figure 1 illustrates these comparisons between organization types. The thick and solid horizontal lines represent E(r) values for a closed organization, while the thick and dashed lines are E(r) values for an open organization when the two organization types differ. Various sorting costs per agent are shown by thin and solid horizontal lines.

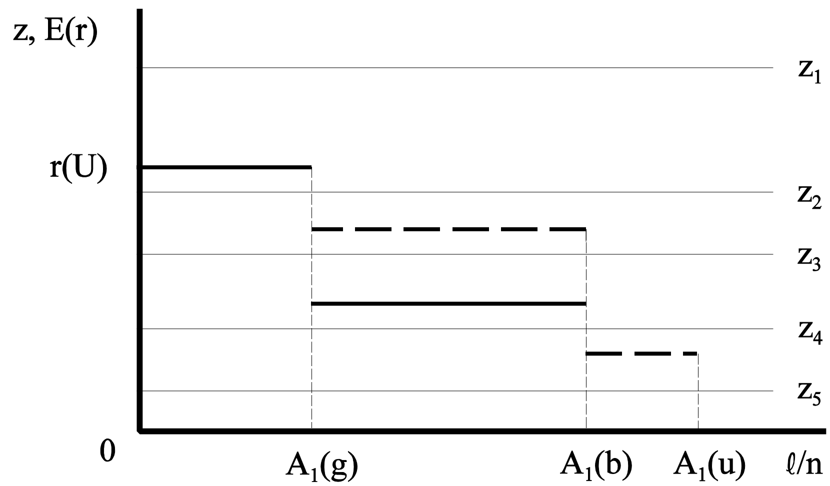

Figure 1

If z exceeds r(U), as z_1 does in Figure 1, then in equilibrium neither type of organization sorts any agents, and Result 1 applies. If the sorting cost is like z_2, then the only possible equilibrium for both open and closed organizations is one in which R/n = 1 < $A_1(g)$. That is, an equilibrium exists only if $A_1(g)$ happens to be larger than 1. This case corresponds to Result 2. The result follows because if $A_1(g)$ # 1 then agents behave either as G and U, or all types behave as G. However, since z > E(r) the organization will not sort any agents. Hence all agents will behave as U, and no equilibrium exists. This is in accord with the Corollaries to Propositions 3 and 5. The case when z is like z_3 is the same as the case of z_2 except that no equilibrium exists for the closed organization when 1 $ $A_1(g)$, and no equilibrium exists for both organization types if 1 $ $A_1(b)$. If z is like z_4, then the equilibrium depends on where 1 lies on the range from 0 up to (but not including) $A_1(b)$. If 1 < $A_1(g)$, this case also corresponds to Result 2. If $A_1(g)$ # 1 < $A_1(b)$, the implications are as in Result 3. If 1 $ $A_1(b)$, then no equilibrium exists in accord with the Corollaries to Propositions 3 and 5 again. If z is like z_5, the analysis is the same as the case of z_4 except that no equilibrium exists for the closed organization when 1 $ $A_1(b)$, and no equilibrium exists for both organization types when 1 $ $A_1(u)$.

Suppose now that the Rate of Behavioral Substitution is sufficiently large. By reasoning similarly to the analysis above we obtain the following results.[12]

Result 4. In equilibrium, when $z < E(r)$, and the Rate of Behavioral Substitution is sufficiently large, a closed organization is unambiguously inefficient when $1 < A_2(g)$, or when $A_2(u) \# 1 < A_1(g)$ in an open organization and $A_2(b) \# 1 < A_1(g)$ in a closed organization, or when $A_1(g) \# 1 < A_1(b)$ and $1 < A_3(g)$, or when $A_1(b) \# 1 < A_1(u)$ and $1 < A_3(t_i)$, for every t_i 0 {g,b}, or when $1 \$ A_1(u)$ and $1 < A_3(t_i)$, for every t_i 0 {g,b}. This is so because even though both organization types obtain the same outcome and recoup the same penalties, the closed organization expends resources screening agents before they are admitted to the organization.

Result 5. In equilibrium, when $z < E(r)$, and the Rate of Behavioral Substitution is sufficiently large, a closed organization may or may not be inefficient when $A_2(g) \# 1 < A_2(b)$, or when $A_2(b) \# 1 < A_2(u)$ in an open organization and $A_2(b) \# 1 < A_1(g)$ in a closed organization, or when $A_1(g) \# 1 < A_1(b)$ and $1 \$ A_3(g)$. This is so because even though the closed organization expects a larger outcome, it expends resources screening agents before they are admitted to the organization and also expects to recoup less in penalties than the open organization.

Results 1 - 5 can be used to formulate the following, arguably testable, implications regarding the choice of organizational type by any institutional entity.

(i) Organizations facing sufficiently high screening costs (because the screening cost per agent is high or because the frequency of intrinsically ugly types in the population is high) will choose the open type. This follows from all the results. If all agent types choose the same behavioral type regardless of organization type, Results 1, 2 and 4 imply that the open type is more efficient regardless of screening cost. If different agent types choose different behavioral types, Results 3 and 5 imply that open organizations are more efficient provided that screening costs are sufficiently high.

(ii) Organizations facing sufficiently high sorting costs relative to penalties recouped for agent misbehavior will choose the open type. This follows from Result 1.

[12] An appendix containing this analysis is available from the authors upon request. Also note that a figure analogous to Figure 1 could be drawn to illustrate these comparisons between organization types when the Rate of Behavioral Substitution is sufficiently large.

(iii) Organizations facing a sufficiently high frequency of intrinsically good types in the population or a sufficiently low frequency of intrinsically ugly types will choose the open type. This follows from all the results. If all agent types choose the same behavioral type regardless of organization type, Results 1, 2 and 4 imply that the open type is more efficient regardless of the frequency of good or ugly types. If different agent types choose different behavioral types, Results 3 and 5 imply that open organizations are more efficient provided the frequency of good (ugly) types is sufficiently high (low).

(iv) Organizations facing a sufficiently high outcome from agents who behave as good or bad relative to the outcome received from agents who behave as ugly will choose the closed organization type, provided that the sorting cost is sufficiently low. This follows from Results 3 and 5.

(v) Organizations that can recoup sufficiently high penalties for bad or ugly behavior will choose the open type, provided that the sorting cost is sufficiently low. If all agent types choose the same behavioral type regardless of organization type, Results 2 and 4 imply that the open type is more efficient regardless of penalties. If different agent types choose different behavioral types, Results 3 and 5 imply that open organizations are more efficient, provided the penalties are sufficiently high.

As mentioned above, the preceding results assume that the sorting cost per agent either exceeds the penalty expected to be received from any sorted agent in both open and closed organizations, or it does not exceed the expected penalty in both organizations. Assume now that both organization types face the same sorting cost per agent, but this cost exceeds the penalty expected to be received from any sorted agent in one organization type but not in the other type for the same range of possible R/n values. Then clearly this can occur only when the distribution of agent behavior differs across organization types. However, when the distribution of agent behavior differs, the expected penalty is always larger for the open organization. Therefore, if the sorting cost exceeds the expected penalty for one organization type only, it must be the closed type. This was discussed in passing in the preceding analysis of this section. Two examples are shown by lines z_3 (when $A_1(g) \# 1 < A_1(b)$) and z_5 (when $A_1(b) \# 1 < A_1(u)$) in Figure 1. The Corollary to Proposition 5 demonstrates that there is no equilibrium for the closed organization in this case. Hence we cannot compare the organizational types when this occurs.

In the preceding analysis, we also assumed that organizations of all types draw agents from the same pool. This assumption allowed us to infer that there are more intrinsically good and bad types in a closed organization than in an open organization (condition (2.8)). The implication is that closed organizations expect a larger outcome and less penalties to be recouped in the cases analyzed in Results 3 and 5. However, if the different types of organization draw agents from different pools, then we can never tell a priori how the distributions of intrinsic types differ in the two organization types, hence we cannot compare agent behavior (and hence expected outcome and penalties) in the two organizations at all. As an example, suppose that an open organization draws agents from a pool in which there is a plethora of intrinsically good types, and the closed organization draws agents from a pool in which good types are scarce. Then the likelihood of good types in a closed organization (even though it screens all agents *ex ante*) may be smaller than that of an open organization. One important implication of this observation for our analysis, and for any empirical research, is that we must be careful to determine whether agents are drawn from the same pool or not. If we are comparing organizational differences in, say, firms in the same industry and in the same geographical area, then it probably is a safe assumption that they are drawing agents from the same pool. However, if we are comparing US versus European or Japanese firms, they may be drawing agents from different pools, hence, comparing agent behavior may be impractical.

So far we have focused on efficiency from the organization's perspective. Here we extend the analysis to social efficiency. Clearly in cases where we cannot determine which organizational type is efficient from the organization's perspective, we cannot determine which type is socially efficient either. The cases analyzed in Results 3 and 5 fall into this category. In the remaining cases, that is, those presented in Results 1, 2 and 4, closed organizations are socially inefficient as well as being organizationally inefficient, only when all agent types behave as U. This is so because closed organizations deny entry to agents of type u who are replaced by g and b types. Since the u types would face no behavioral adjustment cost, while the g and b types do, closed organizations are less efficient both from the organization's perspective and from the agents' perspective. However, when all agent types behave as B in equilibrium, replacing u types with g and b types in closed organizations leads to lower behavioral adjustment costs, because the b types face no adjustment costs.[13] Hence, closed organizations may or may not be socially inefficient. Thus our next set of results is:

[13] Note that agents of type u face a "one-step" adjustment cost, similar to g type agents, while b type agents face no adjustment costs, which in this case is advantageous to a closed organization.

Result 6. In equilibrium, when $z \geq r(U)$, or when $z < r(U)$, $1 < A_1(g)$ and the Rate of Behavioral Substitution is sufficiently small, or when $z < r(U)$, $1 < A_2(g)$ and the Rate of Behavioral Substitution is sufficiently large, a closed organization is unambiguously socially inefficient. This is so because both organization types sort no agents and all agents behave as ugly, but the closed organization expends resources screening agents before they are admitted to the organization and the agents face higher behavioral adjustment costs.

Result 7. In equilibrium, when $z < E(r)$, $A_1(g) \leq 1 < A_1(b)$ and the Rate of Behavioral Substitution is sufficiently small, or when $z < E(r)$, the Rate of Behavioral Substitution is sufficiently large, and $A_2(u) \leq 1 < A_1(g)$ in an open organization and $A_2(b) \leq 1 < A_1(g)$ in a closed organization, or $A_1(g) \leq 1 < A_1(b)$ and $1 < A_3(g)$, or $A_1(b) \leq 1 < A_1(u)$ and $1 < A_3(t_i)$, for every $t_i \in \{g,b\}$, or $1 \geq A_1(u)$ and $1 < A_3(t_i)$, for every $t_i \in \{g,b\}$, or when $z < E(r)$, the Rate of Behavioral Substitution is sufficiently large, and $A_2(g) \leq 1 < A_2(b)$, or $A_2(b) \leq 1 < A_2(u)$ in an open organization and $A_2(b) \leq 1 < A_1(g)$ in a closed organization, or $A_1(g) \leq 1 < A_1(b)$ and $1 \geq A_3(g)$, a closed organization may or may not be socially inefficient. In some cases this is so because both organization types obtain the same outcome and recoup the same penalties, and agents in closed organizations face lower behavioral adjustment costs, but the closed organization expends resources screening agents before they are admitted to the organization. In other cases this is so because the closed organization expects a larger outcome than the open organization, and agents in the closed organization may face lower behavioral adjustment costs, but the closed organization expends resources screening agents before they are admitted to the organization and also expects to recoup less in penalties.

6. Extensions

The remaining analysis extends our results by relaxing some of our assumptions. Specifically, first we allow the average sorting cost to vary with the number of agents sorted; second we study the case when the organization has the power to precommit to a sorting frequency; and third we briefly consider sequential rather than simultaneous sorting.

6A. Economies and Diseconomies of Scale in Sorting Costs

The first extension we consider is average sorting costs that depend on the number of agents sorted. Even though we focused on the case of a constant average cost of sorting, z, our analysis applies much more generally. A constant average cost of sorting does imply that the organization will sort either all agents or none, but this would also be the case with economies of scale in the sorting cost. This is so because if it is worth sorting $0 < R < n$ agents (i.e., if the expected benefit per agent from collecting a penalty outweighs the sorting cost per agent when $0 < R < n$ agents are sorted), then it is worth sorting all n agents, which corresponds to all cases except (i) in Propositions 3 and 5. In addition, if it is not worth sorting all n agents, then it is not worth sorting any $0 < R < n$ agents. This corresponds to cases (i) in Propositions 3 and 5.

We now turn to the case in which diseconomies of scale are present. For ease of exposition, we assume there is a continuum of agents, and therefore the marginal sorting cost function is continuous in R/n. It can then be shown that an equilibrium always exists at the R/n such that the marginal cost of sorting equals the marginal benefit E(r).

The rationale for this result is as follows. Lemmas 2 and 4 show that for R/n values in different intervals agents will choose specific behavioral types, which lead to specific outcomes X that are observed by the organization. As shown previously, once the organization observes X it can infer exactly the number of agents behaving as each type and hence expects a unique penalty per sorted agent, E(r). If agents expect the organization to choose an R in a particular interval determined by the $A_1(\cdot)$ and $A_2(\cdot)$ values, such that the marginal cost of sorting equals the E(r) in that interval, then they behave in a way such that the X observed by the organization will lead the organization to expect the same E(r) as above. The organization will then choose the same R that agents expected. Note, however, that E(r) is a step function; hence, if the marginal sorting cost function crosses E(r) at a point where E(r) is discontinuous, no equilibrium exists. We demonstrate this by example in Figure 2, where we use the case of the closed organization when the Rate of Behavioral Substitution is small (see Table 3). Let the total sorting cost be $Z = z(R)R$. In Figure 2, the Z' curves are different marginal sorting cost functions, and $\Lambda(U)$ is the true frequency of agents behaving as U. E(r) can be r(U) (when all agents behave as ugly), or $\Lambda(U)$r(U) (when only intrinsically bad types behave as ugly), or 0 (when all types behave as good).

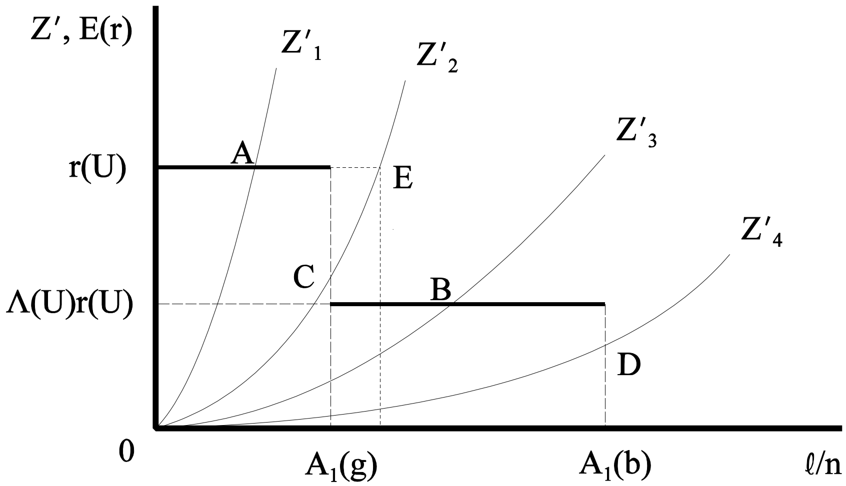

Figure 2

Clearly points A and B are equilibria in Figure 2. However, when ZN crosses the E(r) function where it is discontinuous, such as points C and D, there is no equilibrium. For instance, at C or any point to the right of C on ZN$_2$ there is no equilibrium because agents expect R/n $ A$_1$(g) > 0, and therefore the b types behave as U and the g types behave as G. But if the organization expects E(r) = Λ(U)r(U), it chooses R/n = 0 because ZN$_2$ at A$_1$(g) exceeds E(r). At any point to the left of C on ZN$_2$, agents expect A$_1$(g) > R/n $ 0, and hence all agents behave as U. But if the organization expects E(r) = r(U), it chooses the R/n corresponding to point E, where R/n > A$_1$(g).[14] Thus there is no equilibrium when the marginal sorting cost is ZN$_2$.

As argued earlier, E(r) for the closed organization is always smaller or equal to that for the open organization, at the same R/n. Therefore closed organizations will never sort more agents than open organizations, if both organizations sort agents in equilibrium. This is shown by points A and B in Figure 3 where the solid line represents E(r) for the closed organization and the horizontal dashed line depicts E(r) for the open organization. The organizational efficiency implications are that closed organizations bear screening costs but less sorting costs, and expect to enjoy a larger outcome but

[14] It is straightforward to show that if there is no continuum of agents but, instead, the number of agents is discrete, then the equilibrium occurs at the largest R/n at which ZN # E(r). Note that if ZN > E(r) for R = 1, then the organization will not sort any agents in equilibrium.

recoup less in penalties. Further, similar to the constant sorting cost case, if Z' exceeds E(r) in one organization type, it must be the closed type. Then, no equilibrium exists for the closed organization, and hence no comparison of efficiency between organization types is possible. An example is shown by points C and D in Figure 3.

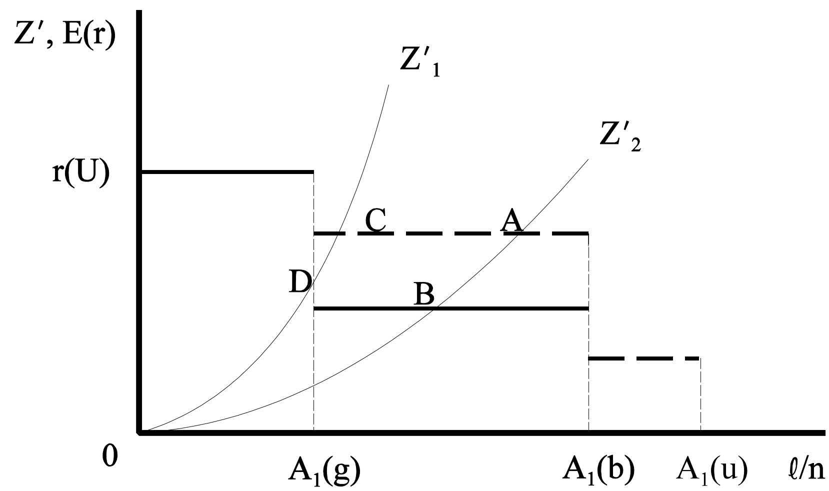

Figure 3

6B. Precommitment by the Organization

Next, we turn to the case when the organization has the power to precommit to a sorting frequency, assuming the cost per agent sorted is constant. The equilibrium of the game in this case is characterized by: (i) a strategy R for the organization determining the number of agents to be sorted; and (ii) a choice of behavioral type τ_i by each intrinsic type $t_i \in \{g,b,u\}$ of agent, $\tau_i(t_i|R) \in \{G,B,U\}$. The agents' strategies and the organization's strategy and beliefs must satisfy the following conditions. First, the organization chooses the strategy that maximizes its expected payoff shown in (6.1) for an open organization, or in (6.2) for a closed organization, given the agents' responses to the organization's choice:

(6.1)
$$n \sum_{t_i} \left[p(t_i) \left[x_i(\tau_i(t_i|R)) + \frac{R}{n}\left[-z + r(\tau_i(t_i|R))\right] \right] \right],$$

$$(6.2) \qquad -E(m)s + n \sum_{t_i} \left[\mu(t_i)\left[x_i(\tau_i(t_i|R)) + \frac{R}{n}\left[-z + r(\tau_i(t_i|R))\right]\right]\right],$$

where E(m) = n/[p(g) + p(b)] is the expected number of agents that have to be screened by the closed organization to get n good and bad agents. This is so because the number of good and bad agents obtained through screening follows a binomial distribution. Second, an agent's choice of behavioral type depending on his intrinsic type, and given the precommitted sorting frequency R/n, maximizes his expected payoff in (2.1).

Clearly, under precommitment, all equilibria in Propositions 3 and 5 where $z < E(r)$ and $R = n$ survive. This is so because if $z < E(r)$, then R should equal n even with precommitment to R. In these cases, precommitment has no value because it does not affect the equilibrium behavior of agents compared to the no-precommitment case. However, when $z \geq r(U)$, the equilibrium may differ from case (i) in Propositions 3 and 5 in which $R = 0$. With precommitment, the organization has to trade off the net cost of sorting (sorting cost minus penalties recouped) against the benefit from better agent behavior (higher output, X) and may sort $0 < R \leq n$ agents in equilibrium. Since the trade-off may or may not differ across organization types, they may or may not sort the same number of agents. The rationale for this is that even though closed organizations engage in *ex ante* screening, there may still be scope for extensive sorting to encourage good behavior. Precommitting to a low sorting frequency would invite opportunistic misbehavior by the agents. As a example we consider the small Rate of Behavioral Substitution case, where agents behave as either G or U in equilibrium, to show that, in fact, open organizations may sort the same or more agents than closed organizations. Refer to Figure 1 for the analysis below.

For ease of exposition, we assume there is a continuum of agents. We also assume the $A_1(\cdot)$ values are small in the sense that $1 > A_1(u)$ for generality (if they were large, the analysis would be simpler). Clearly, to minimize the loss from sorting, the organization will always sort the minimum number of agents necessary to induce the agent behavior it wants to implement. For example, if it is optimal for both organization types to implement good behavior by all agent types, then the open organization will sort exactly $A_1(u)$ agents, and the closed organization will sort exactly $A_1(b)$ agents. Thus an open organization chooses the sorting frequency among 0, $A_1(g)$, $A_1(b)$, or $A_1(u)$ that maximizes (6.1), while the closed organization chooses among 0, $A_1(g)$ or $A_1(b)$ to maximize (6.2). It is easy to see that precommitment now may have value in both organization types, because the equilibrium can occur at $R > 0$. For instance, if x(G) is relatively large in the sense that $x(G) > x(U) + A_1(b)z$, then $R = 0$ will never be an equilibrium in either type of organization.

Finally, while no equilibrium exists in the no precommitment case when E(r) # z < r(U) as shown in the Corollaries to Propositions 3 and 5, there is always an equilibrium under precommitment in which the organization selects R to maximize (6.1) or (6.2), and the agents behave accordingly.

6C. Sequential Sorting

We now briefly show how the analysis can be extended when we allow for sequential instead of simultaneous sorting, keeping the cost per agent sorted constant. First note that if z $ E(r), and if an equilibrium exists, no sorting will occur in equilibrium even if sorting is sequential. Thus we focus on the case where z < E(r). We showed in the simultaneous sorting case that if an equilibrium with sorting exists the organization sorts all agents, R = n. We now characterize the optimal number of agents to be sorted when sorting is done sequentially. Specifically, we characterize the optimal stopping rule. For ease of exposition, we focus on the case where agents behave as either G or U in equilibrium. As argued in the preceding analysis, once the organization observes X, it can infer the actual number of agents behaving as G, $n\Lambda(G)$, and as U, $n\Lambda(U)$. Recall that, in this case, the organization can recoup a penalty from an agent only if it sorts that agent and finds him to be ugly. After R agents have been sorted and U(R) agents have been found to be ugly, the organization will not find it optimal to sort one more agent if

(6.3)
$$\frac{n\Lambda(U) \,\&\, U(R)}{n \,\&\, R}\, r(U) \,\#\, z\,,$$

where $[n\Lambda(U) - U(R)] / [n - R]$ is the probability that the next agent sorted is found to be ugly.[15] The actual R may be less than in the simultaneous sorting case if the organization finds a high proportion of the uglies early in the sorting process, in which case it is not worth sorting additional agents because the probability that the next agent sorted is found to be ugly is low.[16] In this setting, the agent's choice of behavioral type in equilibrium is best response to his expectation of R, and the organization's choice of R is best response to X in accord with (6.3). Comparing the equilibria in the two organization types and the associated organizational efficiency, when R is a random variable due to the sequential sorting, is the subject of future work. We refer the reader to the literature on stochastic games with stopping.

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[15] Note that the sequence U(1),U(2),...,U(R),... is a *submartingale* if E[U(R)] exists because with probability 1 E[U(R+1)] $ U(R).

[16] It is theoretically possible that R = n if n - 1 uglies were found in R - 1 sequential sortings. The organization then knows in advance that the last agent to be sorted will be found ugly, and it does sort him when z < r(U).

7. Conclusions

This paper develops a novel model of agent behavior in two stylized types of organization, open and closed, that differ in the degree to which each scrutinizes potential affiliates. An open organization does no screening of agents before they are admitted to the organization, while a closed organization screens all agents prior to admitting them. After observing the aggregate outcome, both organization types have the option to engage in *ex post* "auditing" of agent behavior (called sorting in our model) and penalize agents whose behavior is subpar. Agents can be of different intrinsic types (good, bad and ugly in our model) that differ in the degree to which they value misbehavior. Actual agent behavior (called behaviorally good, bad and ugly) depends on the short term net benefit of that behavior versus the expected penalty if caught misbehaving. The model is general enough to allow the organization to be a variety of institutions. For example, the organization could be a firm and the agents potential employees, or the organization could be a country and the agents potential immigrants, or a school or licensing authority dealing with applicants. The focus of the paper is agent behavior in these organization types and the associated efficiency implications.

One might expect *a priori* that closed organizations are more efficient than open organizations because one would anticipate better agent behavior and less equilibrium sorting, given that the closed organization screens agents and denies entry to the worst types. Surprisingly, this is not the case. Less sorting by the organization would invite opportunistic misbehavior by agents, and thus there would be a trade-off between payoffs to the organization and the costs of screening and sorting. We show that under quite general conditions regarding the sorting costs, in particular when the sorting costs per agent are constant or declining because of economies of scale, the closed organization will engage in the same amount of sorting as the open organization. Specifically, either all agents are sorted or none are sorted in equilibrium. Agents of the same intrinsic type, expecting the same amount of sorting, will behave identically in the two organization types. When agent behavior is uniform across intrinsic types, closed organizations turn out to be inefficient because they engage in costly *ex ante* screening without any improvement in agent behavior. If agent behavior differs across intrinsic types, then closed organizations end up with better agent behavior and thus may or may not be inefficient.

Not surprisingly, agent behavior is uniformly ugly if the organization does no screening. However, when agents are screened and all agent types behave uniformly, either all behave as ugly or all behave as bad. All behave as ugly when even the good intrinsic types find it "profitable" to misbehave because the short-term net benefit of behaving as ugly outweighs the penalty when sorted. Interestingly, agent behavior in equilibrium will never be uniformly good. This is so because if all agents behaved as good the organization would not want to sort any agents, in which case all agents would behave as ugly. A prerequisite for this result is that the organization cannot precommit to a sorting frequency.

When we extend the analysis to the precommitment case, we find that precommitment may or may not have value. Specifically, precommitment has no value when the sorting cost is less than the penalties expected to be recouped through sorting, because the organization will still find it optimal to sort all agents. By contrast, precommitment has value when there are equilibria under precommitment while no equilibria exist under no-precommitment, and in the case in which the sorting cost exceeds the penalties expected to be recouped, because instead of sorting no agents, the organization may precommit to sorting some agents at a loss in order to induce better behavior. We conjecture that closed organizations may sort less agents in equilibrium than open organizations, in order to induce the same agent behavior.

We also extend the analysis to allow for decreasing returns to scale in sorting, or for sequential rather than simultaneous sorting. In both cases we find that the number of sorted agents can be intermediate; that is, other than all or none. Under decreasing returns to scale we also find that closed organizations will never sort more agents in equilibrium than open organizations, and when the organizations sort a different number of agents, closed organizations induce better agent behavior even though they sort fewer agents.

We raised two main questions in the introduction: First, how does organization type affect agent behavior and organization payoff, and second, what factors determine the choice of organizational type? We answer the first question thoroughly as summarized above. Answers to the second question rely heavily on our analysis of the efficiency of different organization types. If all agent types behave as ugly or all as bad in equilibrium, then any organization will choose to be open to avoid the screening costs. If agent behavior is a mixture of types, then the choice of organization type depends on the trade-off between payoffs to the organization and the costs of screening and sorting.

Our main analysis yields the following implications. Organizations facing sufficiently high screening costs or, interestingly, sufficiently high sorting costs per agent relative to penalties recouped for agent misbehavior, will choose the open organization type. If the organization believes that the probability of ugly intrinsic types is sufficiently high or sufficiently low or the probability of good intrinsic types is sufficiently high, it will choose the open organization type again. Organizations facing a sufficiently high outcome from agents who behave as good or bad relative to the outcome received from agents who behave as ugly will choose the closed organization type, provided that the sorting cost is sufficiently low. Organizations that can recoup sufficiently high penalties for bad or ugly behavior will choose the open type, provided that the sorting cost is sufficiently low.

To conclude, our analysis shows that, overall, open organizations are more efficient than closed when all agent types behave uniformly across organization types. Open organizations are also socially efficient when all agents behave as ugly. However, when agent behavior is richer, either type of organization can be efficient under the right circumstances.

Appendix

Proof of Lemma 1. Given Assumptions 1b, 2 and 4, it can be shown that (3.8) and (3.11) hold. Given Assumptions 1a, 1b and 4, it follows that

$$A_1(g) < A_1(b) < A_1(u), \tag{A1}$$

and

$$A_2(g) \# A_2(b) < A_2(u). \tag{A2}$$

Suppose that $RBS_{BU}(t_i)$ satisfies condition (3.7). Given this condition and Assumptions 1a, 1b and 4, it follows that $A_1(u) < A_2(g)$. This finding, and conditions (A1) and (A2) above, complete the proof of (3.9). Note that (3.9) is consistent with condition (3.5), that is, given that (3.7) and (3.8) imply that $RBS_{BU}(t_i) < r(B) / r(U)$, œ t_i 0 {g,b,u}, it follows that $A_1(t_i) < A_2(t_i)$, œ t_i 0 {g,b,u}.

Suppose now that $RBS_{BU}(t_i)$ satisfies condition (3.10). Given this condition and Assumptions 1a, 1b, and 4, it follows that $A_2(u) < A_1(g)$. This finding, and conditions (A1) and (A2) above, complete the proof of (3.12). Note that (3.12) is consistent with condition (3.5), that is, given that (3.10) and (3.11) imply that $RBS_{BU}(t_i) > r(B) / r(U)$, œ t_i 0 {g,b,u}, it follows that $A_1(t_i) > A_2(t_i)$, œ t_i 0 {g,b,u}.

QED

Proof of Lemma 2. Assume that condition (3.7) holds. Condition (3.5) then implies that $A_1(t_i) < A_2(t_i)$ for every t_i. Starting from any intrinsic type t_i 0 {u,b,g}, the agent must decide whether to behave as U, B or G with the following expected payoffs:

$$E(U*t_i) = v(U) - k(U*t_i) - \frac{R}{n} r(U), \tag{A3}$$

$$E(B*t_i) = v(B) - k(B*t_i) - \frac{R}{n} r(B), \tag{A4}$$

$$E(G*t_i) = v(G) - k(G*t_i). \tag{A5}$$

It can easily be verified that the agent will never behave as B, because $E(B*t_i) > E(U*t_i)$ and $E(B*t_i) > E(G*t_i)$ contradicts (3.5). Given Assumption 5, the agent will behave as U if

(A6) $$E(U{*}t_i) > E(B{*}t_i)$$

and

(A7) $$E(U{*}t_i) > E(G{*}t_i).$$

By rearranging terms, it can easily be shown that condition (A6) is equivalent to

(A8) $$\frac{R}{n} < A_2(t_i).$$

Similarly, condition (A7) is equivalent to

(A9) $$\frac{R}{n} < A_1(t_i).$$

However, since $A_1(t_i) < A_2(t_i)$, it follows that the agent will behave as U if (A9) is satisfied (in which case (A8) is automatically satisfied). This fully proves (3.13a) and partially proves (3.13b) and (3.13c).

Given Assumption 5, the agent will behave as G if

(A10) $$E(G{*}t_i) \ \$ \ E(B{*}t_i)$$

and

(A11) $$E(G{*}t_i) \ \$ \ E(U{*}t_i).$$

By rearranging terms, it can be shown that condition (A10) is equivalent to

(A12) $$\frac{R}{n} \ \$ \ A_1(t_i) \ \frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}.$$

Opposite to (A9), condition (A11) is equivalent to

(A13) $$\frac{R}{n} \ \$ \ A_1(t_i).$$

It follows that the agent will behave as G if (A13) is satisfied (in which case (A12) is automatically satisfied because (3.7) implies that $RBS_{BU}(t_i)$ is smaller than $r(B) / r(U)$). The findings above along with (3.9) in Lemma 1 complete the proof of (3.13b) and (3.13c) and prove (3.13d).

Assume that condition (3.10) holds. Condition (3.5) then implies that $A_1(t_i) > A_2(t_i)$ for every t_i. Starting from any intrinsic type $t_i \in \{u,b,g\}$, the agent must decide whether to behave as U, B or G with expected payoffs as in (A3), (A4) and (A5) above.

The agent will behave as U if (A6) and (A7) hold, which are equivalent to (A8) and (A9). However, since $A_1(t_i) > A_2(t_i)$, it follows that the agent will behave as U if (A8) is satisfied (in which case (A9) is automatically satisfied). This fully proves (3.14a) and partially proves (3.14b) and (3.14c).

Given Assumption 5, the agent will behave as B if

(A14) $$E(B|t_i) > E(G|t_i)$$

and

(A15) $$E(B|t_i) \geq E(U|t_i).$$

By rearranging terms, it can be shown that condition (A14) is equivalent to

(A16) $$\frac{R}{n} < A_1(t_i) \frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}.$$

Condition (A15) is equivalent to

(A17) $$\frac{R}{n} \geq A_2(t_i).$$

It follows that the agent will behave as B if

(A18) $$A_2(t_i) \leq \frac{R}{n} < A_1(t_i) \frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}.$$

Note that (3.10) and (3.11) imply $\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}} > 1, \forall t_i$. This along with (A18) and (3.12) in Lemma

1 complete the proof of (3.14b) and (3.14c), prove (3.14d), and partially prove (3.14e), (3.14f) and (3.14g).

Given Assumption 5, the agent will behave as G if (A10) and (A11) hold, which are equivalent to (A12) and (A13). It follows that the agent will behave as G if (A12) is satisfied (in which case (A13) is automatically satisfied because (3.10) implies that $RBS_{BU}(t_i) > r(B) / r(U)$). Conditions (A12) and (A18) along with (3.12) in Lemma 1 complete the proof of (3.14e), (3.14f) and (3.14g). QED

Proof of Proposition 3. First note that the organization's assessments of the frequencies of agents behaving as any of the behavioral types after observing aggregate outcome X , $\Phi(\tau_i{*}X) = \varphi(\tau_i{*}X)/n$, are calculated in accordance with the discussion following the characterization of the equilibrium at the end of section 2. Then the expected penalty E(r) is calculated in accordance with these assessments. Recall that penalties satisfy Assumption 4. Assume condition (3.7) holds.

(i) If $z \geq E(r)$, agents know the organization will never find it worthwhile to sort any agents. Given $R(\cdot) = 0$ and Assumptions 1, 2 and 3, all agent types behave as ugly; that is, $\tau_i(t_i) = U, \forall i, \forall t_i$.

(ii) If $z < E(r)$ and $A_1(g) > 1$, then at the largest possible $R(\cdot)$ (i.e., at $R(\cdot) = n$), it follows that $R(\cdot) = n < n\,A_1(g)$. Therefore, $R(\cdot) / n < A_1(g), \forall R(\cdot)$. Condition (3.13a) in Lemma 2 implies that $\tau_i(t_i) = U, \forall i, \forall t_i$. Given this behavior, and since $z < E(r)$, the organization will sort $R(\cdot) = n$ agents.

(iii) If $z < E(r)$, $A_1(g) \leq 1$ and $A_1(b) > 1$, then it follows that $n\,A_1(g) \leq R(\cdot) = n < n\,A_1(b)$. Condition (3.13b) in Lemma 2 implies that $\tau_i(g) = G, \forall i$, and $\tau_i(t_i) = U, \forall i, \forall t_i \in \{b,u\}$. Given this behavior, and since $z < E(r)$, the organization will sort $R(\cdot) = n$ agents.

(iv) If $z < E(r)$, $A_1(b) \leq 1$ and $A_1(u) > 1$, then it follows that $n\,A_1(b) \leq R(\cdot) = n < n\,A_1(u)$. Condition (3.13c) in Lemma 2 implies that $\tau_i(t_i) = G, \forall i, \forall t_i \in \{g,b\}$, and $\tau_i(u) = U, \forall i$. Given this behavior, and since $z < E(r)$, the organization will sort $R(\cdot) = n$ agents.

The proof of the case when (3.7) holds is completed by noting that no PBE exists when $A_1(u) \leq 1$. This is so because when $A_1(u) \leq 1$ there are two cases: either $R(\cdot)/n \geq A_1(u)$ or $R(\cdot)/n < A_1(u)$. In the former case, it follows from (3.13d) that $\tau_i(t_i) = G, \forall i, \forall t_i$. Given this behavior, $R(\cdot) = 0$. But if $R(\cdot) = 0$, then $\tau_i(t_i) = U, \forall i, \forall t_i$. Thus there is no PBE. In the latter case it follows that $R(\cdot) < n$. Further, $\tau_i(t_i)$ depends on how much smaller $R(\cdot)/n$ is relative to $A_1(u)$. The three possibilities are given by conditions (3.13a), (3.13b) and (3.13c) in Lemma 2. Given that $z < E(r)$, it is optimal for the organization to sort all agents; that is, $R(\cdot) = n$. Thus there is no PBE in this case either. To conclude, no PBE exists in pure strategies when $A_1(u) \leq 1$.

Assume now that condition (3.10) holds. The proof of case (i) is identical to that when (3.7) holds and thus is omitted. The proof of cases (ii) - (v) is analogous to cases (ii) - (iv) when (3.7) holds except that conditions (3.14a) - (3.14d) in Lemma 2 are used rather than (3.13a) - (3.13c), so that the critical parameters are primarily the $A_2(t_i)$ values. Thus whereas the choice before was between U and G, now it is between U and B. The proof for cases (vi) - (xiii) uses conditions (3.14e) - (3.14g) in Lemma 2. Note that condition (3.16) is used in cases (viii) - (xiii). The proof is analogous to those above except that the critical parameters are now the $A_1(t_i)$ and $A_3(t_i)$ values (the latter depending on $RBS_{BU}(t_i)$); hence, the agents' behavioral choice is between B and G.

The proof of the case when (3.10) holds is completed by noting that no PBE exists when $A_1(u)$ # 1 and $A_3(t_i)$ # 1, œt_i. The reasoning is similar to that when (3.7) holds and $A_1(u)$ # 1. QED.

Proof of Corollary to Proposition 3. Given any putative equilibrium in which E(r) # z < r(U), it follows that R(X) = 0 regardless of X. But if agents expect R(·) = 0, then $\tau_i(t_i)$ = U œi, œt_i; that is, all agents behave as U. However, the condition that E(r) # z < r(U) rules out putative equilibria in which every agent behaves as U, because E(r) would equal r(U) in that case.[17] QED

[17] However, recall that there can be an equilibrium in which every agent behaves as U and R(X) = 0 if z $ r(U), which is case (i) in Proposition 3.

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The Good, the Bad and the Ugly:
Agent Behavior and Efficiency in Open and Closed Organizations

Duncan M. Holthausen
Theofanis Tsoulouhas

Department of Economics
North Carolina State University
Raleigh, NC 27695-8110

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Abstract: This paper develops a novel model of agent behavior in organizations in order to compare the efficiency of "open" versus "closed" organizations. Closed organizations "screen" potential agents before admitting them while open organizations do not. Both have the option to "sort" (audit) individual behavior after observing aggregate outcome. Each agent is intrinsically "good," "bad" or "ugly," but can behave as any of the three types. Screening allows the organization to deny entry to the worst agent types, while sorting allows the organization to penalize opportunistic misbehavior by the agents. We show that both organizations may sort in equilibrium. When the sorting cost per agent is constant or exhibits economies of scale, surprisingly, both organizations sort the same number of agents, which leads agents of the same type to behave uniformly across organizations. However, agent behavior across intrinsic types may or may not be uniform. Interestingly, there is no equilibrium in which all agent types behave as good. When all agent types behave as bad or all behave as ugly, and an equilibrium exists in both organizations, closed organizations are less efficient from the organization's perspective than open ones. When all agent types behave as ugly, closed organizations are socially inefficient as well. If agent behavior is a mixture of types, then closed organizations can be efficient because they screen out some of the worst agent types in advance. When organizations can precommit to a sorting frequency, more equilibria exist; for instance, all agent types may behave as good.

1. Introduction

Why are some organizations or societies more open than other, even seemingly similar, organizations or societies? For example, firms in the same industry, equally developed countries or institutions engaged in the same activities often differ in how selective they are when admitting agents. In addition to this initial screening, organizations frequently monitor, audit or sort their agents *ex post*. Why do some organizations sort more intensively than others, and is there a link between the degree of screening and the intensity of sorting? How does the extent of screening and sorting affect the behavioral choices of agents and the ensuing efficiency of organizations?

We develop a novel framework in this paper to address these important questions and shed light on the actual practice of organizations. We provide an explanation based on the characteristics of the agent populations, the expected equilibrium behavior of agents, the costs of screening and sorting, and the penalty structure set by the institutional environment. The model is general enough to allow the organization to be a variety of institutions. The organization could be a firm and the agents potential employees, or a country and the agents potential immigrants, or a governmental agency enforcing laws, or a school or licensing authority dealing with applicants. For example, big city law and consulting firms are reputedly more lenient in hiring than small city firms but fewer hires make partner. Large Japanese corporations generally provide long-term employment to their employees, but in doing so they are very selective when they hire people. Some companies give tests to all applicants, for example Microsoft reputedly tests the intelligence and creativity of applicants regardless of credentials, other firms such as Home Depot require all potential employees to take drug tests, and banks routinely obtain credit reports for all job applicants. Some occupations require extensive licensure testing. Some universities admit most students who apply but flunk out a large percentage, while others are very selective but graduate most students who matriculate. It is more difficult to get into a respectable Japanese university than a comparable U.S. university, but reputedly easier to graduate. Several European countries test drivers more thoroughly before granting drivers' licenses than in the United States, but monitoring of good driving behavior (for instance, speeding) is less thorough. In many European countries government job applicants are required to present more documents (for instance criminal records) and undergo greater testing than those in the United States, but there may be less scrutiny once the applicants are hired.

Sorting can take different forms. Organizations may monitor agents while the agents are executing tasks or audit agents after they have taken actions. Examples include observation of worker performance by supervisors, periodic performance evaluations, enforcement of traffic laws, tenure decisions at universities, admission to partnership in law and accounting firms, and school testing. According to Cross (2001), many companies routinely fire 5-10% of their least productive workers over the course of a year. Companies use relative performance evaluation and compare employees to averages or to other employees (as in a tournament) to identify laggards and weed out these weak links. In some companies or institutions sorting takes the form of up-or-out contracts. While some companies reform, retrain or reassign the bottom ranked workers, other companies simply fire these employees immediately after they are identified. For instance, in the last few years, GE has instituted a program it calls "Organizational Vitality" in which bottom ranked workers are reformed successfully or they are ousted. Cisco has a plan where the bottom ranked 5% employees are put on a "Performance Improvement Plan." Employees who fail to achieve prespecified milestones are simply "PIPped." On the other hand, Siebel routinely turns over the lowest performing employees without spending resources to revitalize them. Tenure denial at academic institutions (in rates that differ among institutions) is another example.

We develop the model of agent behavior in organizations in section 2. Each agent is intrinsically "good," "bad" or "ugly," but can behave as any of the three types in equilibrium, as shown in sections 3 and 4. Organizations can be "open" or "closed." Closed organizations "screen" potential agents before admitting them while open organizations do not.. Both types of organization have the option to audit or "sort" individual behavior after observing the aggregate outcome obtained by the agents, and impose disciplinary penalties. The initial screening is designed to weed out those agents whose performance is likely to be unacceptable. In contrast, *ex post* sorting aims at isolating agents whose actual behavior is unacceptable, and one might expect that more thorough screening would reduce the scope for sorting. We show that extensive initial screening does not eliminate the scope for *ex post* sorting, because limited sorting invites opportunistic misbehavior by agents; and in equilibrium either both organizations engage in *ex post* sorting or neither one does. Surprisingly, when the sorting cost per agent is constant or exhibits economies of scale, both organizations sort the same number of agents in equilibrium, which leads agents of the same type to behave uniformly across organizations. However, agent behavior across intrinsic types may or may not be uniform. Interestingly, there is no equilibrium in which all agent types behave as good, provided that organizations do not precommit to a sorting frequency. If all agent types behaved as good, organizations would never sort agents. Agents, expecting no sorting, would never behave as good. When all agent types behave as bad or all behave as ugly, and an equilibrium exists in both organizations, closed organizations are less efficient from the organization's perspective than open ones, as shown in section 5. This follows because both organizations sort the same number of agents, so the screening costs incurred by the closed organization are unnecessary. When all agent types behave as ugly, closed organizations are socially inefficient as well. If agent behavior is a mixture of types, then closed organizations can be efficient because they screen out some of the worst agent types in advance.

Our analysis yields the following implications. Organizations facing sufficiently high screening costs or, interestingly, sufficiently high sorting costs per agent relative to penalties recouped for agent misbehavior, will choose the open organization type. If the organization believes that the proportion of ugly intrinsic types in the population from which agents are drawn is sufficiently high or sufficiently low, or the proportion of good intrinsic types is sufficiently high, it will choose the open organization type again. Organizations facing a sufficiently high outcome from agents who behave as good or bad relative to the outcome received from agents who behave as ugly will chose the closed organization type, provided that the sorting cost is sufficiently low. Lastly, organizations that can recoup sufficiently high penalties for bad or ugly behavior will choose the open type, provided that the sorting cost is sufficiently low.

We show in section 6 that most of the findings above extend to the case in which organizations can precommit to a sorting frequency. However, more equilibria are then possible; that is, equilibria exist in cases where no equilibria exist without precommitment. For instance, agent behavior in equilibrium can be uniformly good with precommitment. Thus, precommitment may or may not have value. Precommitment has no value when the sorting cost is less than the penalties expected to be recouped through sorting, because the organization will still find it optimal to sort the same number of agents. By contrast, precommitment has value when there are equilibria under precommitment while no equilibria exist under no-precommitment, and in the case in which the sorting cost exceeds the penalties expected to be recouped, because instead of sorting no agents, the organization may precommit to sorting some agents at a loss in order to induce better behavior.

We also extend the analysis to allow for decreasing returns to scale in sorting, or for sequential rather than simultaneous sorting. In both cases we find that the number of sorted agents can be intermediate; that is, other than all or none, which was the case with constant or increasing returns to scale and simultaneous sorting. Under decreasing returns to scale we also find that closed organizations will never sort more agents in equilibrium than open organizations, and when the organizations sort a different number of agents, closed organizations induce better agent behavior even though they sort fewer agents.

While this paper does not draw directly from existing literature, it is related to other work on auditing and agent behavior in organizations. For instance, Kahlil (1997) examines a principal-agent model in which the principal can audit the agent's compliance with a contract but cannot precommit to auditing. He showed that lack of commitment to auditing when information is asymmetric can lead to production above the level that would be obtained with full information in order to reduce the probability of agent noncompliance. Maskin and Tirole (2001) consider "accountable" democratic systems in which government officials are screened and disciplined by voters and "unaccountable" systems in which government officials are appointed and hence neither screened nor disciplined by voters. Their interest is in determining the circumstances under which each type of system works best.

2. Model

We develop a model with one organization and a finite number of agents. Agents are born with *intrinsic* types but then select *behavioral* types that need not correspond to their intrinsic types. An intrinsic type does not reflect innate ability or competence but rather a predisposition to, for example, work hard, behave legally or conform to organizational norms. The agents make endogenous decisions about their behavioral types by considering the benefits and costs of these decisions. The costs include the expenses associated with adopting various behavioral types contingent on the intrinsic types, and the penalty from being caught (sorted) weighted by its likelihood. Assume three intrinsic types: *intrinsically good, intrinsically bad and intrinsically ugly*; also assume three behavioral types: *behaviorally good, behaviorally bad and behaviorally ugly*. We consider two types of organization, *closed organization* and *open organization*. A closed organization screens its agents before they are admitted into the organization more extensively than does an open organization. For simplicity, we assume a closed organization screens all agents before the agents are admitted to the organization, however, an open organization does no screening at all so all agents are allowed to join. Both a closed and an open organization sort their agents at random after the agents select behavioral types.[1]

The timing of events is as follows: First, nature selects an intrinsic type $t_i \in \{g,b,u\}$ for each agent i, where g stands for intrinsically good, b for intrinsically bad and u for intrinsically ugly, with probability $p_i(t_i) > 0$ and *anonymity*, that is $p_i(t_i) = p(t_i)$, for all i. Then agents privately learn their intrinsic types. Second, the closed organization screens agents at a fixed cost of s per agent, rejecting those who are found to be the worst type, ugly, so that only n intrinsically good and bad types are let

[1] Note that for concreteness we model sorting as taking place once after the agents choose behavioral types. The model, however, could easily be extended to include sorting while behavioral types are being adopted (i.e., monitoring of agents' activities) without changing the results qualitatively.

in.[2] Assuming that the closed organization lets good and bad types in makes the analysis more interesting than if the organization only lets in good types (in which case all agents would behave the same way in equilibrium). The open organization, by contrast, does no screening and n agents, who can be intrinsically good, bad or ugly, are let in. Third, each agent i chooses a behavioral type or action $\tau_i \in \{G,B,U\}$, where G = behaviorally good, B = behaviorally bad and U = behaviorally ugly, at an adjustment cost of $k(\tau_i * t_i)$; that is, the cost of adopting a behavioral type depends on the agent's intrinsic type. Let $\tau = (\tau_1,...,\tau_n)$ be the vector of behavioral types adopted by the agents. Each action τ_i leads deterministically to a payoff to the agent of $v(\tau_i)$ and to an outcome $x_i(\tau_i)$ for the organization, with $x_i(U) = x(U)$, $x_i(B) = x(B)$, $x_i(G) = x(G)$, $\forall i$, and $x(G) > x(B) > x(U) > 0$. Fourth, even though the organization does not observe individual outcomes, it does observe the aggregate outcome,[3] $X = \sum_{i=1}^{n} x_i(\tau_i)$. Fifth, the organization sorts R agents, and R/n is the probability (frequency) that an agent is sorted. When an agent is sorted, his true behavioral type is publicly revealed. The administrative cost to sort an agent is assumed to be the same for both types of organization and is fixed and equal to z. An agent who is found to be τ_i pays a predetermined penalty $r(\tau_i)$. We assume the penalties are predetermined because the legal system, standard industry practices, organizational norms and outside agencies such as accrediting or overseeing bodies commonly predetermine or restrict the penalties for various types of behavior. Our analysis applies to these cases. Depending on the organization, the penalty could take many forms; for example, it could be a reduction in salary or a fine for illegal behavior. For simplicity we assume that the penalty imposed on the agents is paid to the organization. Qualitatively similar results would be obtained if the payoff to the organization did not coincide with the penalty but was systematically dependent on it. For instance, if a firm determines that an employee should be reformed by taking more extensive training, the employee suffers a welfare loss while the firm enjoys an increase in productivity, both dependent on the amount of training.

We make the following assumptions:

Assumption 1a. $k(G*g) = k(B*b) = k(U*u) = 0$.

1b. $k(2) = k(G*u) = k(U*g) \geq$

[2] For simplicity, we assume that organizations can determine through screening whether agents are intrinsically ugly, but determining precisely whether agents are intrinsically good or bad is prohibitively costly. In a more complex model we could endogenize the quality of screening by the closed organization. We conjecture that the results would not change qualitatively.

[3] Note that the model can easily be expanded to include the moral hazard case in which the organization observes individual contributions, but those are stochastically dependent on unobservable actions taken by the agents.

$$\$\ 2k(1) = 2k(B*u) = 2k(U*b) = 2k(G*b) = 2k(B*g) > 0.$$

Assumption 1a means there is no cost to the agent in adopting a good (bad) (ugly) behavioral type when his intrinsic type is also good (bad) (ugly). We assume in 1b that the cost of choosing a behavioral type depends on whether the agent chooses a behavioral type one or two steps removed from his intrinsic type. For instance an agent who is intrinsically good moves one step if he chooses to behave as bad and moves two steps if he chooses to behave as ugly. The rational for the symmetry in the behavioral adjustment cost structure is that the direction of adjustment does not matter because these are psychic or learning costs. For example, an agent who is born a hard worker will find it as difficult to adjust to shirking as it is for a born shirker to become a hard worker. An honest agent may also find it as difficult to adopt illegal behavior as it is for a criminal to adjust to living honestly. If an agent chooses a behavioral type which is two steps removed from his intrinsic type, then we assume that the cost k(2) is at least twice the cost k(1) of choosing a behavioral type which is only one step removed from his intrinsic type. Hence the behavioral adjustment cost is assumed to follow a "convex" pattern.

Assumption 2. $v(B) - v(G) > v(U) - v(B) > 0, v(G) > 0.$

Assumption 2 ensures that the payoff to the agent, excluding type adjustment costs k(·) and penalties r(·), follows a "strictly concave" pattern. This assumption implies v(U) > v(B) > v(G) > 0. For example, the payoff from shirking is higher than that from working hard because, even though the monetary reward may be less, the cost of effort is much less than that from working hard.

Assumption 3. $v(U) - k(U*b) - v(B) > 0$ and $v(U) - k(U*g) - v(G) > 0.$

Assumption 3 means that without *ex post* sorting, and hence penalties, starting from b (or g) an agent prefers to behave as U rather than as B (or G).[4] For example, absent police enforcement, many people would engage in illegal activities. This assumption makes the analysis more interesting because it means agents have an inherent incentive for misbehavior and thus there may be scope for sorting.

Assumption 4. $r(U)\ \$\ 2r(B)$ and $r(B) > r(G) = 0.$

[4] Recall that k(B*b) = k(G*g) = 0.

Assumption 4 means that the penalty imposed on agents when found behaviorally ugly is at least twice as high as that when found bad, which in turn is higher than the penalty (normalized to zero) imposed when found good.[5] This ensures that the penalty structure is "convex."

If an agent of intrinsic type t_i chooses behavioral type τ_i, his expected payoff given his adjustment cost and, if sorted, his penalty is

$$E(\tau_i|t_i) = v(\tau_i) - k(\tau_i|t_i) - \frac{R}{n} r(\tau_i). \tag{2.1}$$

The open organization's expected payoff after observing the aggregate outcome X is

$$\Pi_o(R|\tau,X) = \sum_{i=1}^{n} x_i(\tau_i) + R\left[-z + \sum_{\tau_i} \Phi(\tau_i|X)\, r(\tau_i)\right], \tag{2.2}$$

where $\Phi(\tau_i|X) = \varphi(\tau_i|X)/n$ is the organization's assessment of the frequency of agents behaving as τ_i, made after observing X, with $\varphi(\tau_i|X)$ being the assessment of the number of agents behaving as τ_i. The assessed frequency $\Phi(\tau_i|X)$ is given by Bayes rule

$$\Phi(\tau_i|X) = \frac{\Phi(X|\tau_i)\, \Phi(\tau_i)}{\sum_{\tau_i} \Phi(X|\tau_i)\, \Phi(\tau_i)}, \tag{2.3}$$

where,

$$\Phi(\tau_i) = \sum_{t_i} p(\tau_i|t_i)\, p(t_i). \tag{2.4}$$

Note that in pure strategies, the probability $p(\tau_i|t_i)$ that agent i chooses action τ_i given his intrinsic type t_i, which enters (2.4), equals either 1 or 0.

In a closed organization, the expected payoff after observing the aggregate outcome X is

$$\Pi_c(R|\tau,X) = -\,ms + \sum_{i=1}^{n} x_i(\tau_i) + R\left[-z + \sum_{\tau_i} \Phi(\tau_i|X)\, r(\tau_i)\right], \tag{2.5}$$

[5] Because the penalties are predetermined, the organization can utilize only the number of agents to be sorted to induce good behavior. As the analysis below will indicate, this is not sufficient to induce good behavior by all agents, therefore we cannot include incentive compatibility constraints that ensure good behavior by all agents. Note that the analysis directly generalizes to predetermined rewards as opposed to penalties.

where m is the number of agents that had to be screened to find n good and bad agents, $\Phi(\tau_i | X)$ is determined by Bayes rule as above, and

$$(2.6) \qquad \Phi(\tau_i) = \sum_{t_i} p(\tau_i | t_i)\, \mu(t_i),$$

with the probability of agent i being of type t_i in a closed organization given by

$$(2.7) \qquad \mu(t_i) = \frac{p(t_i)}{p(g) + p(b)}, \ \forall t_i \in \{g,b\}.$$[6]

The rationale for (2.7) is that the closed organization screens the agents and removes all the intrinsically ugly types u. Note that this implies

$$(2.8) \qquad \mu(t_i) > p(t_i), \ \forall t_i \in \{g,b\},$$

because agents of type u are replaced with agents of types g and b. Of course, if the closed organization draws from a different population than does the open organization (e.g., a European versus a US pool), it is not possible to compare the distributions of the intrinsic types in the two organizations (that is, neither (2.7) nor (2.8) apply in this case).

In analyzing the extensive-form game played between the organization and the agents, given the type of organization, we adopt perfect Bayesian equilibrium (PBE) as the solution concept and we focus on pure strategy equilibria. The equilibrium of the game is characterized by: (i) a choice of behavioral type τ_i by each intrinsic type t_i of agent, that is, $\tau_i(g)$, $\tau_i(b)$ and $\tau_i(u)$, with $\tau_i(t_i) \in \{G,B,U\}$; (ii) a function $\Phi(\tau_i | X) \in [0,1]$ denoting the organization's beliefs about agent i's behavioral type after observing the aggregate outcome X; (iii) a strategy $R(X)$ for the organization determining the number of agents to be sorted based on the observed value of aggregate outcome X. The agents' strategies and the organization's strategy and beliefs must satisfy the following conditions. First, an agent's choice of behavioral type given his intrinsic type is sequentially rational, that is, it maximizes his expected payoff in (2.1), given the organization's strategy and belief function. Second, given the agents' strategies, the organization's observation of aggregate outcome and its belief function, the organization

[6] Note that for ease of exposition we have abused the notation; for instance, we use the same notation for all $\Phi(\cdot)$ probabilities regardless of the type of organization, even though the probabilities depend on the type of organization.

chooses the strategy that maximizes its expected payoff shown in (2.2) for an open organization, or in (2.5) for a closed organization. Third, the organization's belief function is derived from Bayes rule according to (2.3).

The following cases are possible regarding belief probabilities about behavioral types: **(i)** If X = nx(G) or X = nx(U), the organization receives a fully informative signal about each agent's behavioral type because when X = nx(G) the organization knows that $\tau_i(t_i)$ = G œi, and when X = nx(U), $\tau_i(t_i)$ = U œi. **(ii)** If X = nx(B), either all agents behave as bad or the average output obtained by agents behaving as good, φ(G*X), and those who behave as ugly, φ(U*X), is x(B), regardless of the number who behave as bad, φ(B*X). **(iii)** If nx(U) < X < nx(G) and agents behave in equilibrium as one of two types, then the organization infers the number of agents behaving as either type exactly by solving two linear equations with two unknowns. For instance, suppose agents behave as good or ugly, then φ(G*X) + φ(U*X) = n and φ(G*X)x(G) + φ(U*X)x(U) = X. If agents behave in equilibrium as one of three types, then determining the exact number who behave as each type is obviously not possible.

3. Agent Behavior in An Open Organization

We start by defining a number of parameters that will be useful in characterizing behavior in equilibrium. First, let $RBS_{BU}(t_i)$ denote the Rate of Behavioral Substitution defined as the ratio of the marginal benefit of behaving as B rather than as G divided by the marginal benefit of behaving as U rather than as G, excluding penalties if sorted and starting from any intrinsic type t_i. Thus,

(3.1) $$RBS_{BU}(t_i) = \frac{[v(B) \& k(B*t_i)] \& [v(G) \& k(G*t_i)]}{[v(U) \& k(U*t_i)] \& [v(G) \& k(G*t_i)]}.$$

The analysis below will demonstrate the importance of this rate in our results. The rationale is that agents find it tempting to adopt bad or ugly behavior rather than good; hence, equilibrium properties depend on the rate of substitution between bad and ugly behavior.

Agent behavior will also be shown to depend on whether the sorting frequency R/n lies in various intervals defined by the two parameters defined below. Let

(3.2) $$A_1(t_i) = \frac{[v(U) \& k(U*t_i)] \& [v(G) \& k(G*t_i)]}{r(U)},$$

and

$$(3.3) \qquad A_2(t_i) = \frac{[v(U) \& k(U^*t_i)] \& [v(B) \& k(B^*t_i)]}{r(U) \& r(B)}.$$

$A_1(t_i)$ shows the marginal benefit to the agent of behaving as U rather than as G, divided by the marginal penalty imposed on the agent if he is sorted and found to have behaved as U rather than as G (recall that r(G) = 0 which simplifies the denominator of (3.2)). $A_2(t_i)$ shows the marginal benefit to the agent of behaving as U rather than as B, divided by the marginal penalty imposed on the agent if he is sorted and found to have behaved as U rather than as B. It can easily be shown that $RBS_{BU}(t_i)$ is related to $A_1(t_i)$ and $A_2(t_i)$. In particular, for every t_i,

$$(3.4) \qquad RBS_{BU}(t_i) = 1 - \frac{A_2(t_i)}{A_1(t_i)} \frac{[r(U) \& r(B)]}{r(U)}.$$

Condition (3.4) implies that

$$(3.5) \qquad RBS_{BU}(t_i) < (=) (>) \frac{r(B)}{r(U)} \;]\; A_1(t_i) < (=) (>) A_2(t_i),$$

which means that if the Rate of Behavioral Substitution between B and U, rather than G, is smaller (larger) than the ratio of penalties if sorted, then the marginal benefit of behaving as U rather than as G, divided by the corresponding marginal penalty if sorted, is smaller (larger) than the marginal benefit of behaving as U rather than as B, divided by the marginal penalty if sorted.

For an agent of intrinsic type t_i, denote the marginal benefit of behaving as U rather than G as

$$(3.6) \qquad D(t_i) = [v(U) - k(U^*t_i)] - [v(G) - k(G^*t_i)] = A_1(t_i)\, r(U).$$

Note that given Assumptions 2 and 1b, it follows that D(b) > 0, and given Assumption 1a, 1b and 2 it follows that D(u) > 0, which will be useful in Lemma 1. Lemma 1 ranks the $A_1(t_i)$ and $A_2(t_i)$ values for all intrinsic types depending on the magnitude of the Rate of Behavioral Substitution.

Lemma 1. Given Assumptions 1, 2 and 4, in an open organization, if $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U), that is,

$$(3.7) \qquad RBS_{BU}(t_i) < \frac{r(B)}{r(U)} \& \frac{2r(U)[k(2)\&k(1)] \& r(B)k(2)}{D(b)r(U)} \;,\text{œ}t_i,$$

with

$$\frac{2r(U)[k(2)-k(1)] - r(B)k(2)}{D(b)r(U)} > 0, \tag{3.8}$$

then

$$A_1(g) < A_1(b) < A_1(u) < A_2(g) \leq A_2(b) < A_2(u), \tag{3.9}$$

and if $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U), that is,

$$RBS_{BU}(t_i) > \frac{r(B)}{r(U)} + \frac{2k(2)}{D(u)} \frac{[r(U) - r(B)]}{r(U)}, \forall t_i, \tag{3.10}$$

with

$$\frac{2k(2)}{D(u)} \frac{[r(U) - r(B)]}{r(U)} > 0, \tag{3.11}$$

then

$$A_2(g) \leq A_2(b) < A_2(u) < A_1(g) < A_1(b) < A_1(u). \tag{3.12}$$

Proof. See Appendix.

Given the significance of Lemma 1 for the remaining analysis, further discussion is warranted. We focus on the polar cases where r(B) / r(U) is larger than the RBSs for all intrinsic types, or smaller than the RBSs for all intrinsic types. Cases in between can also be analyzed, but they lead to unnecessary complexity without adding any interesting results. Condition (3.7) requires that $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U). The rate is sufficiently low when the payoff structure $v(\cdot)$ is not very "concave" so that condition (3.7) holds (i.e., the agents are not very risk-averse with respect to payoffs). This can easily be seen by fixing the denominator in $RBS_{BU}(t_i)$, in which case the condition is satisfied if the numerator is small. Condition (3.10) requires that $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U). The rate is sufficiently high when the payoff structure $v(\cdot)$ is sufficiently "concave" so that condition (3.10) holds.[7]

Lemma 2 below shows that agent behavior depends on whether the sorting frequency R/n lies in various intervals defined by the $A_1(t_i)$ and $A_2(t_i)$ parameters in Lemma 1. To characterize tie breaking cases we make the following regularity assumption.

Assumption 5. If $\tilde{\tau} = U$ and $\hat{\tau}$ 0 {G,B} or if $\tilde{\tau} = B$ and $\hat{\tau} = G$, then $\tau_i(t_i) = \tilde{\tau}$ iff $E(\tau_i = \tilde{\tau} * t_i) > E(\tau_i = \hat{\tau} * t_i)$. If $\tilde{\tau} = G$ and $\hat{\tau}$ 0 {B,U} or if $\tilde{\tau} = B$ and $\hat{\tau} = U$, then $\tau_i(t_i) = \tilde{\tau}$ iff $E(\tau_i = \tilde{\tau} * t_i)$ \$ $E(\tau_i = \hat{\tau} * t_i)$.

This assumption means that if an agent is indifferent between two distinct behavioral types, he will adopt the "better" behavioral type. The agent adopts a "worse" behavioral type only when his payoff is strictly larger for the worse type. For instance, the agent of some given intrinsic type will behave as U rather than as B if the payoff for behaving as U is strictly greater than the payoff for behaving as B. If the payoffs are the same, the agent will behave as B.

Lemma 2. Given Assumption 5 and Lemma 1, in an open organization, if condition (3.7) holds so that $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U), then for any 0 # R/n #1:

(3.13a) If $\frac{R}{n} < A_1(g)$, then $\tau_i(t_i) = U$, œi, œt_i;

(3.13b) if $A_1(g)$ # $\frac{R}{n} < A_1(b)$, then $\tau_i(g) = G$, œi, and $\tau_i(t_i) = U$, œi, œt_i.0 {b,u};

(3.13c) if $A_1(b)$ # $\frac{R}{n} < A_1(u)$, then $\tau_i(t_i) = G$, œi, œt_i 0 {g,b}, and $\tau_i(u) = U$, œi;

(3.13d) if $A_1(u)$ # $\frac{R}{n}$, then $\tau_i(t_i) = G$, œi, œt_i.

Given Assumption 5 and Lemma 1, in an open organization, if condition (3.10) holds so that $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U), then for any 0 # R/n #1:

(3.14a) If $\frac{R}{n} < A_2(g)$, then $\tau_i(t_i) = U$, œi, œt_i;

[7] By using equation (3.4) it can be shown that condition (3.10) is satisfied iff [v(B) - v(G)] > [v(U) - v(B)] + 4k(1), which supports the intuition that the payoff structure needs to be sufficiently "concave" (i.e., that the agents are sufficiently risk-averse with respect to payoffs).

(3.14b) if $A_2(g) \leq \frac{R}{n} < A_2(b)$, then $\tau_i(g) = B, \forall i$, and $\tau_i(t_i) = U, \forall i, \forall t_i \in \{b,u\}$;

(3.14c) if $A_2(b) \leq \frac{R}{n} < A_2(u)$, then $\tau_i(t_i) = B, \forall i, \forall t_i \in \{g,b\}$, and $\tau_i(u) = U, \forall i$;

(3.14d) if $A_2(u) \leq \frac{R}{n} < A_1(g)$, then $\tau_i(t_i) = B, \forall i, \forall t_i$;

(3.14e) if $A_1(g) \leq \frac{R}{n} < A_1(b)$, then $\tau_i(t_i) = B, \forall i, \forall t_i \in \{b,u\}$, and

$$\text{if } \frac{R}{n} < A_1(g)\frac{RBS_{BU}(g)}{\frac{r(B)}{r(U)}}, \text{ then } \tau_i(g) = B, \forall i, \text{ but}$$

$$\text{if } \frac{R}{n} \geq A_1(g)\frac{RBS_{BU}(g)}{\frac{r(B)}{r(U)}}, \text{ then } \tau_i(g) = G, \forall i;$$

(3.14f) if $A_1(b) \leq \frac{R}{n} < A_1(u)$, then $\tau_i(u) = B, \forall i$, and

$$\text{if } \frac{R}{n} < A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, \ t_i \in \{g,b\}, \text{ then } \tau_i(t_i) = B, \forall i, \forall t_i \in \{g,b\}, \text{ but}$$

$$\text{if } \frac{R}{n} \geq A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, \ t_i \in \{g,b\}, \text{ then } \tau_i(t_i) = G, \forall i, \forall t_i \in \{g,b\};$$

(3.14g) if $A_1(u) \leq \frac{R}{n}$, then

$$\text{if } \frac{R}{n} < A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, \ t_i \in \{g,b,u\}, \text{ then } \tau_i(t_i) = B, \forall i, \forall t_i, \text{ but}$$

$$\text{if } \frac{R}{n} \geq A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, \ t_i \in \{g,b,u\}, \text{ then } \tau_i(t_i) = G, \forall i, \forall t_i.$$

Proof. See Appendix.

The behavior of agents can be summarized in the following tables. To simplify notation, let

$$(3.15) \qquad A_3(t_i) = A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}} = \frac{[v(B) \& k(B{*}t_i)] \& [v(G) \& k(G{*}t_i)]}{r(B)}.$$

$A_3(t_i)$ reflects the marginal benefit of behaving as B rather than as G relative to the marginal cost. It is useful for the remaining analyses to note that Assumptions 1, 2 and 3 imply

$$(3.16) \qquad A_3(g) < A_3(b) \# A_3(u).$$

		R/n < $A_1(g)$	R/n < $A_1(b)$	R/n < $A_1(u)$	R/n \$ $A_1(u)$
t_i	g	U	G	G	G
	b	U	U	G	G
	u	U	U	U	G

Table 1. Behavioral type chosen by agent in an open organization when $RBS_{BU}(\cdot)$ is small.

						R/n < $A_1(b)$		R/n < $A_1(u)$		R/n \$ $A_1(u)$	
		R/n < $A_2(g)$	R/n < $A_2(b)$	R/n < $A_2(u)$	R/n < $A_1(g)$	R/n< $A_3(g)$	R/n\$ $A_3(g)$	R/n< $A_3(t_i)$ t_i 0 {g,b}	R/n\$ $A_3(t_i)$ t_i 0 {g,b}	R/n< $A_3(t_i)$ œ t_i	R/n\$ $A_3(t_i)$ œ t_i
t_i	g	U	B	B	B	B	G	B	G	B	G
	b	U	U	B	B	B	B	B	G	B	G
	u	U	U	U	B	B	B	B	B	B	G

Table 2. Behavioral type chosen by agent in an open organization when $RBS_{BU}(\cdot)$ is large.

Observe that for any given value of R/n, at most only two behavioral types are chosen by agents (U and G when $RBS_{BU}(\cdot)$ is small and U and B or G and B when $RBS_{BU}(\cdot)$ is large). In the small $RBS_{BU}(\cdot)$ case behavior is extreme, specifically agents adopt only ugly or good behavior. The intuition for this result is that when $RBS_{BU}(\cdot)$ is small, agents are not very risk averse with respect to payoffs, as argued in the discussion of Lemma 1. Therefore, they are willing to accept the risk of behaving as ugly provided that the probability of being caught ($\mathcal{R}/n$) is low. However, when the chance of being caught is high, they adopt good behavior. Interestingly, they never behave as bad when the chance of getting caught is high, because a relatively small $RBS_{BU}(\cdot)$ is indicative of a small marginal benefit of behaving as bad rather than as good.

Behavior in the large $RBS_{BU}(\cdot)$ case is richer. In this case, the marginal benefit of behaving as bad rather than as good is more substantial and agents are more risk averse than in the small $RBS_{BU}(\cdot)$ case. Therefore the agents behave as ugly only if the probability of being caught is very low. If the probability is intermediate in value, agents tend to behave as bad to reduce the penalty they face if they are sorted. If the probability of being caught is high, agents behave as bad or good. They behave as bad if the probability of being caught is not very high, given that the marginal benefit of behaving as bad rather than as good is large. By contrast, if the probability of being caught is very high, they behave as good.

Given the findings in Lemma 2, Proposition 3 characterizes the equilibrium of the extensive game played between the open organization and the agents. The proposition highlights the importance of the sorting cost z relative to the expected penalty recouped from the sorted agents. It also emphasizes the significance of the $A_1(t_i)$, $A_2(t_i)$ and $A_3(t_i)$ values which, again, reflect the marginal benefit of behaving as U rather than as G, U rather than as B, and B rather than as G, respectively, relative to the penalty imposed if sorted. Proposition 3 is long because of the variety of possible cases. We give the underlying intuition immediately following the proposition, so the reader may skip over the details without losing the essence of the results.

Proposition 3. Let $E(r) = \sum_{\tau_i} \Phi(\tau_i|X)\, r(\tau_i(t_i))$ be the expected penalty from sorting agent i, and let $\Lambda(\tau_i)$ be the true frequency of agents behaving as τ_i. Given Assumption 3 and Lemma 2, in an open organization, if condition (3.7) holds so that $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U), then in the PBE of the extensive-form game the following cases are possible:

Case (i): $z \geq E(r)$

Agent behavior:	$\tau_i(t_i) = U, \forall i, \forall t_i$
Outcome observed:	$X = nx(U)$
Assessments:	$\Phi(U|nx(U)) = \Lambda(U) = 1$
Expected penalty:	$E(r) = r(U), \forall i$

Number of agents sorted: $\mathbb{R}$(nx(U)) = 0

Case (ii): z < E(r) and A_1(g) > 1

Agent behavior: $\tau_i(t_i)$ = U, œi, œt_i

Outcome observed: X = nx(U)

Assessments: Φ(U*nx(U)) = Λ(U) = 1

Expected penalty: E(r) = r(U), œi

Number of agents sorted: $\mathbb{R}$(nx(U)) = n

Case (iii): z < E(r), A_1(g) # 1 and A_1(b) > 1

Agent behavior: τ_i(g) = G, œi, and $\tau_i(t_i)$ = U, œi, œt_i 0 {b,u}

Outcome observed: nx(U) < X = φ(G*X)x(G) + φ(U*X)x(U) < nx(G)

Assessments: Φ(G*X) = Λ(G), Φ(U*X) = Λ(U)

Expected penalty: E(r) = Φ(U*X) r(U), œi

Number of agents sorted: $\mathbb{R}$(X) = n

Case (iv): z < E(r), A_1(b) # 1 and A_1(u) > 1

Agent behavior: $\tau_i(t_i)$ = G, œi, œt_i 0 {g,b} and τ_i(u) = U, œi

Outcome observed: nx(U) < X = φ(G*X)x(G) + φ(U*X)x(U) < nx(G)

Assessments: Φ(G*X) = Λ(G), Φ(U*X) = Λ(U)

Expected penalty: E(r) = Φ(U*X) r(U), œi

Number of agents sorted: $\mathbb{R}$(X) = n

Given Assumption 3 and Lemma 2, in an open organization, if condition (3.10) holds, so that $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U), then in the PBE of the extensive-form game the following cases are possible:

Case (i): z $ E(r)

Agent behavior: $\tau_i(t_i)$ = U, œi, œt_i

Outcome observed: X = nx(U)

Assessments: Φ(U*nx(U)) = Λ(U) = 1

Expected penalty: E(r) = r(U), œi

Number of agents sorted: $\mathbb{R}$(nx(U)) = 0

Case (ii): z < E(r) and A_2(g) > 1

Agent behavior: $\tau_i(t_i)$ = U, œi, œt_i

Outcome observed: X = nx(U)

Assessments: Φ(U*nx(U)) = Λ(U) =1

Expected penalty: E(r) = r(U), œi

Number of agents sorted: R(nx(U)) = n

Case (iii): z < E(r), A_2(g) # 1 and A_2(b) > 1

Agent behavior: $\tau_i(g)$ = B, œi, and $\tau_i(t_i)$ = U, œi, œt_i 0 {b,u}

Outcome observed: nx(U) < X = φ(B*X)x(B) + φ(U*X)x(U) < nx(B)

Assessments: Φ(B*X) = Λ(B), Φ(U*X) = Λ(U)

Expected penalty: E(r) = Φ(B*X) r(B) + Φ(U*X) r(U), œi

Number of agents sorted: R(X) = n

Case (iv): z < E(r), A_2(b) # 1 and A_2(u) >1

Agent behavior: $\tau_i(t_i)$ = B, œi, œt_i 0 {g,b} and $\tau_i(u)$ = U, œi

Outcome observed: nx(U) < X = φ(B*X)x(B) + φ(U*X)x(U) < nx(B)

Assessments: Φ(B*X) = Λ(B), Φ(U*X) = Λ(U)

Expected penalty: E(r) = Φ(B*X) r(B) + Φ(U*X) r(U), œi

Number of agents sorted: R(X) = n

Case (v): z < E(r), A_2(u) # 1 and A_1(g) >1

Agent behavior: $\tau_i(t_i)$ = B, œi, œt_i

Outcome observed: X = nx(B)

Assessments: Φ(B*X) = Λ(B) = 1

Expected penalty: E(r) = r(B), œi

Number of agents sorted: R(nx(B)) = n

Case (vi): z < E(r), A_1(g) # 1, A_3(g) >1,[8] and A_1(b) >1

Agent behavior: $\tau_i(t_i)$ = B, œi, œt_i

Outcome observed: X = nx(B)

Assessments: Φ(B*X) = Λ(B) = 1

Expected penalty: E(r) = r(B), œi

Number of agents sorted: R(nx(B)) = n

Case (vii): z < E(r), A_3(g) # 1 and A_1(b) >1

Agent behavior: $\tau_i(g)$ = G, œi, and $\tau_i(t_i)$ = B, œi, œt_i 0 {b,u}

Outcome observed: nx(B) < X = φ(G*X)x(G) + φ(B*X)x(B) < nx(G)

Assessments: Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)

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[8] Recall that $A_3(t_i)$ was defined in (3.15).

Expected penalty: E(r) = Φ(B*X) r(B), œi

Number of agents sorted: R(X) = n

Case (viii): z < E(r), A_1(b) # 1, A_3(g) >1, and A_1(u) >1

Agent behavior: $\tau_i(t_i)$ = B, œi, œt_i

Outcome observed: X = nx(B)

Assessments: Φ(B*X) = Λ(B) = 1

Expected penalty: E(r) = r(B), œi

Number of agents sorted: R(nx(B)) = n

Case (ix): z < E(r), A_1(b) # 1, A_3(g) # 1, A_3(b) > 1 and A_1(u) >1

Agent behavior: τ_i(g) = G, œi, and $\tau_i(t_i)$ = B, œi, œt_i 0 {b,u}

Outcome observed: nx(B) < X = φ(G*X)x(G) + φ(B*X)x(B) < nx(G)

Assessments: Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)

Expected penalty: E(r) = Φ(B*X) r(B), œi

Number of agents sorted: R(X) = n

Case (x): z < E(r), A_3(b) # 1 and A_1(u) >1

Agent behavior: $\tau_i(t_i)$ = G, œi, œt_i 0 {g,b}, and τ_i(u) = B, œi

Outcome observed: nx(B) < X = φ(G*X)x(G) + φ(B*X)x(B) < nx(G)

Assessments: Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)

Expected penalty: E(r) = Φ(B*X) r(B), œi

Number of agents sorted: R(X) = n

Case (xi): z < E(r), A_1(u) # 1, A_3(g) >1, A_3(b) > 1 and A_3(u) >1

Agent behavior: $\tau_i(t_i)$ = B, œi, œt_i

Outcome observed: X = nx(B)

Assessments: Φ(B*X) = Λ(B) = 1

Expected penalty: E(r) = r(B), œi

Number of agents sorted: R(nx(B)) = n

Case (xii): z < E(r), A_1(u) # 1, A_3(g) # 1, A_3(b) > 1 and A_3(u) >1

Agent behavior: τ_i(g) = G, œi, and $\tau_i(t_i)$ = B, œi, œt_i 0 {b,u}

Outcome observed: nx(B) < X = φ(G*X)x(G) + φ(B*X)x(B) <nx(G)

Assessments: Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)

Expected penalty: E(r) = Φ(B*X) r(B), œi

Number of agents sorted: R(X) = n

Case (xiii): z < E(r), A_1(u) #1, A_3(g) # 1, A_3(b) # 1 and A_3(u) > 1

Agent behavior:	$\tau_i(t_i)$ = G, œi, œt_i 0 {g,b}, and τ_i(u) = B, œi
Outcome observed:	nx(B) < X = φ(G*X)x(G) + φ(B*X)x(B) <nx(G)
Assessments:	Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)
Expected penalty:	E(r) = Φ(B*X) r(B), œi
Number of agents sorted:	R(X) = n

Proof. See Appendix.

The rationale behind Proposition 3 is as follows. In deciding whether to sort agents or not, after observing the aggregate outcome X, the organization compares the sorting cost z to the penalties it expects to collect, E(r). Since z is a constant, the organization will sort either all agents or none depending on the relative value of z.[9] When the sorting cost exceeds the expected penalty from sorting each agent, the organization will not sort any agents, and all agents will behave as ugly. This is case (i). By contrast, when E(r) exceeds z, the organization will sort all agents, so that R/n equals 1. Then agent behavior in equilibrium depends on whether $A_1(\cdot)$ or $A_2(\cdot)$ are larger or smaller than 1. Agent behavior in equilibrium is consistent with Lemma 2 which is summarized in Tables 1 and 2. When $A_1(\cdot)$ is larger than 1, the marginal benefit of behaving as U rather than as G exceeds the marginal cost if sorted, and agents will prefer to behave as U rather than G. The opposite is true when $A_1(\cdot)$ is smaller than 1. When $A_2(\cdot)$ is larger than 1, the marginal benefit of behaving as U rather than as B exceeds the marginal cost if sorted, and agents will prefer to behave as U rather than B. The opposite is true when $A_2(\cdot)$ is smaller than 1. When $A_3(\cdot)$ is larger than 1, the marginal benefit of behaving as B rather than as G exceeds the marginal cost if sorted, and agents will prefer to behave as B rather than as G.[10]

When the Rate of Behavioral Substitution is sufficiently small, all $A_2(\cdot)$ values are greater than 1 for all intrinsic types, meaning that all agent types prefer U to B, and thus agents basically choose only between U and G, which is consistent with agent behavior as shown in Table 1. If even the good

[9] The analysis below will demonstrate that our results are quite general and apply to cases where z is not constant as well (see the extensions in section 6 below).

[10] In the small $RBS_{BU}(\cdot)$ case, $A_3(\cdot)$ larger than 1 also implies that $A_1(\cdot)$ is larger than 1, so that agents prefer both B and U to G. Clearly, some combinations of $A_1(\cdot)$, $A_2(\cdot)$ and $A_3(\cdot)$ will even lead to complete rankings of behavioral types for the agents. For instance, if $A_1(t_i)$ is larger than 1 and $A_2(t_i)$ is smaller than 1, agents of type t_i will prefer B to U to G. However, complete rankings are not necessary to characterize the equilibrium because all that is needed is a behavioral type which is preferred to any other type.

agents have strong incentives to behave as ugly rather than as good when all agents are sorted, then all agents will behave as ugly. This is case (ii) and corresponds to the first column of results in Table 1. Clearly the other cases occur when bad or ugly agent types have inherent incentives to misbehave, but good types do not. In reference to the last column of Table 1, if $A_1(u)$ # R/n = 1, all agents would behave as G if the organization sorted all agents; however, when all agents behave as G the organization will not sort any agents. Hence, there is no PBE is this case.

When the Rate of Behavioral Substitution is sufficiently large, behavior is more complex. In cases (ii) - (iv), the $A_1(\cdot)$ values are greater than 1 for all intrinsic types, meaning that all agent types prefer U to G, and thus agents basically choose only between U and B. In the remaining cases, $A_2(\cdot)$ is smaller than 1 for all intrinsic types, meaning that all agent types prefer B to U, and thus agents basically choose only between B and G. Note that if $A_1(u)$ # 1 and $A_3(t_i)$ # 1 for all t_i, then all agents would behave as G if the organization sorted all agents; however, when all agents behave as G the organization will not sort any agents. Hence, there is no PBE in this case, which corresponds to the last column in Table 2. In general, the Corollary to Proposition 3 characterizes a condition under which no PBE exists.

Corollary to Proposition 3. Let E(r) be the expected penalty from any putative equilibrium in which E(r) # z < r(U). Then the putative equilibrium is not an equilibrium.

Proof. See Appendix.

The case in which all agents behave as G satisfies the condition in the corollary because E(r) = 0. Thus, again, there is no equilibrium in which all agent types behave as G. In general, the condition E(r) < r(U) is satisfied only when some or all agent types behave as B or G. If, in addition, E(r) # z < r(U), then there is no equilibrium. The intuition is that if in the putative equilibrium E(r) # z < r(U), then the organization would not sort any agents and all agent types would behave as ugly, which contradicts the condition above because E(r) would equal r(U) in the putative equilibrium. Note that the case in which z exceeds E(r), so that the organization does no sorting and all agents behave as ugly, does not fit the condition above because z $ E(r) = r(U) in this putative equilibrium.[11]

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[11] This is equilibrium case (i) in Proposition 3.

4. Agent Behavior in a Closed Organization

Recall that a closed organization screens all agents, rejecting those who are found to be intrinsically ugly so that only n intrinsically good and bad types are let in. Therefore, the analysis of a closed organization is similar to that of an open organization, modified to eliminate u as an acceptable intrinsic type. We focus again on the two polar cases regarding the Rate of Behavioral Substitution analyzed in section 3. In the first case $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U) for all intrinsic types as in condition (3.7). In the second case $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U) for all intrinsic types as in condition (3.10). Lemma 4 below characterizes agent behavior for the two cases depending on the sorting frequency R/n in relation to the $A_1(t_i)$ and $A_2(t_i)$ values.

Lemma 4. Given Assumption 5 and Lemma 1, in a closed organization, if condition (3.7) holds so that $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U), then for any 0 # R/n #1:

(4.1a) If $\frac{R}{n} < A_1(g)$, then $\tau_i(t_i) = U$, œi, œt_i;

(4.1b) if $A_1(g)$ # $\frac{R}{n} < A_1(b)$, then $\tau_i(g) = G$, œi, and $\tau_i(b) = U$, œi;

(4.1c) if $A_1(b)$ # $\frac{R}{n}$, then $\tau_i(t_i) = G$, œi, œt_i.

Given Assumption 5 and Lemma 1, in a closed organization, if condition (3.10) holds so that $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U), then for any 0 # R/n #1:

(4.2a) If $\frac{R}{n} < A_2(g)$, then $\tau_i(t_i) = U$, œi, œt_i;

(4.2b) if $A_2(g)$ # $\frac{R}{n} < A_2(b)$, then $\tau_i(g) = B$, œi, and $\tau_i(b) = U$, œi;

(4.2c) if $A_2(b)$ # $\frac{R}{n} < A_1(g)$, then $\tau_i(t_i) = B$, œi, œt_i;

(4.2d) if $A_1(g)$ # $\frac{R}{n} < A_1(b)$, then $\tau_i(b) = B$, œi, and

if $\frac{R}{n} < A_1(g)\frac{RBS_{BU}(g)}{\frac{r(B)}{r(U)}}$, then $\tau_i(g) = B$, œi, but

$$\text{if } \frac{R}{n} \ \$ \ A_1(g)\frac{RBS_{BU}(g)}{\frac{r(B)}{r(U)}}, \text{ then } \tau_i(g) = G, \text{ œi;}$$

(4.2e) if $A_1(b)$ # $\frac{R}{n}$, then

$$\text{if } \frac{R}{n} < A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, \ t_i \ 0 \ \{g,b\}, \text{ then } \tau_i(t_i) = B, \text{ œi, œ}t_i\text{, but}$$

$$\text{if } \frac{R}{n} \ \$ \ A_1(t_i)\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}, \ t_i \ 0 \ \{g,b\}, \text{ then } \tau_i(t_i) = G, \text{ œi, œ}t_i.$$

Proof. The proof is analogous to the proof of Lemma 2 in the Appendix.

The intuition behind the results is similar to the intuition behind the results in Lemma 2. The only difference is that now there are no intrinsically ugly types in the organization, which reduces the number of cases to be considered. The behavior of agents can be summarized in the following tables. Similar to section 3, to simplify notation we define $A_3(t_i)$ as in (3.15).

		$R/n < A_1(g)$	$R/n < A_1(b)$	R/n \$ $A_1(b)$
t_i	g	U	G	G
	b	U	U	G

Table 3. Behavioral type chosen by agent in a closed organization when $RBS_{BU}(\cdot)$ is small.

		$R/n <$ $A_2(g)$	$R/n <$ $A_2(b)$	$R/n <$ $A_1(g)$	$R/n < A_1(b)$		R/n \$$A_1(b)$	
					$R/n<$ $A_3(t_i)$	R/n\$ $A_3(t_i)$	$R/n<$ $A_3(t_i)$	R/n\$ $A_3(t_i)$
t_i	g	U	B	B	B	G	B	G
	b	U	U	B	B	B	B	G

Table 4. Behavioral type chosen by agent in a closed organization when $RBS_{BU}(\cdot)$ is large.

Proposition 5 characterizes the equilibrium of the extensive game played between the closed organization and the agents. Similar to Proposition 3, Proposition 5 highlights the importance of the sorting cost z relative to the expected penalty recouped from the sorted agents, and the significance of the marginal benefit of behaving as U rather than as G or B, or B rather than as G, relative to the penalty imposed if sorted (i.e., the significance of the $A_1(\cdot)$, $A_2(\cdot)$ and $A_3(\cdot)$ values).

Proposition 5. Let $E(r) = \text{j}_{\tau_i} \Phi(\tau_i*X)\, r(\tau_i(t_i))$ be the expected penalty from sorting agent i, and let $\Lambda(\tau_i)$ be the true frequency of agents behaving as τ_i. Given Assumption 3 and Lemma 4, in a closed organization, if condition (3.7) holds so that $RBS_{BU}(t_i)$ is sufficiently smaller than r(B) / r(U), then in the PBE of the extensive-form game the following cases are possible:

Case (i): z \$ E(r)

Agent behavior:	$\tau_i(t_i) = U$, œi, œt_i
Outcome observed:	$X = nx(U)$
Assessments:	$\Phi(U*nx(U)) = \Lambda(U) = 1$
Expected penalty:	$E(r) = r(U)$, œi
Number of agents sorted:	R(nx(U)) = 0

Case (ii): z < E(r) and $A_1(g) > 1$

Agent behavior:	$\tau_i(t_i) = U$, œi, œt_i
Outcome observed:	$X = nx(U)$
Assessments:	$\Phi(U*nx(U)) = \Lambda(U) = 1$
Expected penalty:	$E(r) = r(U)$, œi
Number of agents sorted:	R(nx(U)) = n

Case (iii): z < E(r), $A_1(g)$ # 1 and $A_1(b) > 1$

Agent behavior:	$\tau_i(g) = G$, œi, and $\tau_i(b) = U$, œi
Outcome observed:	$nx(U) < X = \varphi(G*X)x(G) + \varphi(U*X)x(U) < nx(G)$
Assessments:	$\Phi(G*X) = \Lambda(G)$, $\Phi(U*X) = \Lambda(U)$
Expected penalty:	$E(r) = \Phi(U*X)\, r(U)$, œi
Number of agents sorted:	R(X) = n

Given Assumption 3 and Lemma 4, in a closed organization, if condition (3.10) holds, so that $RBS_{BU}(t_i)$ is sufficiently larger than r(B) / r(U), then in the PBE of the extensive-form game the following cases are possible:

Case (i): z \$ E(r)

Agent behavior: $\tau_i(t_i) = U$, œi, œt_i

Outcome observed: $X = nx(U)$

Assessments: $\Phi(U*nx(U)) = \Lambda(U) = 1$

Expected penalty: $E(r) = r(U)$, œi

Number of agents sorted: R(nx(U)) = 0

Case (ii): z < E(r) and $A_2(g) > 1$

Agent behavior: $\tau_i(t_i) = U$, œi, œt_i

Outcome observed: $X = nx(U)$

Assessments: $\Phi(U*nx(U)) = \Lambda(U) = 1$

Expected penalty: $E(r) = r(U)$, œi

Number of agents sorted: R(nx(U)) = n

Case (iii): z < E(r), $A_2(g)$ # 1 and $A_2(b) > 1$

Agent behavior: $\tau_i(g) = B$, œi, and $\tau_i(b) = U$, œi

Outcome observed: $nx(U) < X = \varphi(B*X)x(B) + \varphi(U*X)x(U) < nx(B)$

Assessments: $\Phi(B*X) = \Lambda(B)$, $\Phi(U*X) = \Lambda(U)$

Expected penalty: $E(r) = \Phi(B*X)\, r(B) + \Phi(U*X)\, r(U)$, œi

Number of agents sorted: R(X) = n

Case (iv): z < E(r), $A_2(b)$ # 1 and $A_1(g) >1$

Agent behavior: $\tau_i(t_i) = B$, œi, œt_i

Outcome observed: $X = nx(B)$

Assessments: $\Phi(B*X) = \Lambda(B) = 1$

Expected penalty: $E(r) = r(B)$, œi

Number of agents sorted: R(nx(B)) = n

Case (v): z < E(r), $A_1(g)$ # 1, $A_3(g) >1$ and $A_1(b) >1$

Agent behavior: $\tau_i(t_i) = B$, œi, œt_i

Outcome observed: $X = nx(B)$

Assessments: $\Phi(B*X) = \Lambda(B) = 1$

Expected penalty: $E(r) = r(B)$, œi

Number of agents sorted: R(nx(B)) = n

Case (vi): z < E(r), $A_3(g)$ # 1 and $A_1(b) >1$

Agent behavior: $\tau_i(g) = G$, œi, and $\tau_i(b) = B$, œi

Outcome observed: $nx(B) < X = \varphi(G*X)x(G) + \varphi(B*X)x(B) < nx(G)$

Assessments: Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)

Expected penalty: E(r) = Φ(B*X) r(B), œi

Number of agents sorted: R(X) = n

Case (vii): z < E(r), A_1(b) # 1, A_3(g) >1

Agent behavior: $\tau_i(t_i)$ = B, œi, œt_i

Outcome observed: X = nx(B)

Assessments: Φ(B*X) = Λ(B) = 1

Expected penalty: E(r) = r(B), œi

Number of agents sorted: R(nx(B)) = n

Case (viii): z < E(r), A_1(b) # 1, A_3(g) # 1 and A_3(b) > 1

Agent behavior: τ_i(g) = G, œi, and τ_i(b) = B, œi

Outcome observed: nx(B) < X = φ(G*X)x(G) + φ(B*X)x(B) < nx(G)

Assessments: Φ(G*X) = Λ(G), Φ(B*X) = Λ(B)

Expected penalty: E(r) = Φ(B*X) r(B), œi

Number of agents sorted: R(X) = n

Proof. The proof is similar to the proof of Proposition 3 in the Appendix.

Similar to the open organization, there are certain cases in which no PBE exists for the closed organization, as the corollary below demonstrates.

Corollary to Proposition 5. Let E(r) be the expected penalty from any putative equilibrium in which E(r) # z < r(U). Then the putative equilibrium is not an equilibrium.

Proof. The proof is identical to the proof of the Corollary to Proposition 3.

5. Open versus Closed Organization

In this section we focus on the efficiency of open versus closed organizations from the organization's perspective, which has implications for the choice of organizational type faced by a would-be organization drawing agents from a given pool. At the end of the section we discuss implications for social efficiency. To analyze organizational efficiency, we use the findings obtained in sections 3 and 4 which are summarized below. We start with cases in which the sorting cost per agent either exceeds the penalty expected to be received from any sorted agent in both open and closed organizations, or does not exceed the expected penalty in both organizations.

In equilibrium, both types of organization sort either no agents or all agents. When the sorting cost per agent, z, exceeds the penalty expected to be received from any sorted agent, E(r), all agents behave as U in both types of organization, regardless of the Rate of Behavioral Substitution, and the organization sorts no agents (R = 0). Thus, our first result regarding efficiency is:

Result 1. In equilibrium, when z $ r(U) in both the open and closed organization, a closed organization is unambiguously inefficient regardless of the Rate of Behavioral Substitution. This is so because both organization types sort no agents and all agents behave as ugly, but the closed organization expends resources screening agents before they are admitted to the organization.

Suppose now that $z < E(r)$, then the organization sorts all agents (R = n, so that R/n = 1). When the Rate of Behavioral Substitution is sufficiently small, agent behavior in equilibrium is summarized in the following tables.

		$1 < A_1(g)$ (a)	$1 < A_1(b)$ (b)	$1 < A_1(u)$ (c)
t_i	g	U	G	G
	b	U	U	G
	u	U	U	U

Table 5. Equilibrium behavior chosen by agent in an open organization when $RBS_{BU}(\cdot)$ is small.

		$1 < A_1(g)$ (a)	$1 < A_1(b)$ (b)
t_i	g	U	G
	b	U	U

Table 6. Equilibrium behavior chosen by agent in a closed organization when $RBS_{BU}(\cdot)$ is small.

And when the Rate of Behavioral Substitution is sufficiently large, agent behavior in equilibrium is summarized in the following tables.

						$1 < A_1(b)$		$1 < A_1(u)$		$1 \$ A_1(u)$
		$1 < A_2(g)$ (a)	$1 < A_2(b)$ (b)	$1 < A_2(u)$ (c1)	$1 < A_1(g)$ (c2)	$1 < A_3(g)$ (d1)	$1 \$ A_3(g)$ (d2)	$1 < A_3(t_i)$ t_i 0 {g,b} (e1)	$1 \$ A_3(t_i)$ t_i 0 {g,b} (e2)	$1 < A_3(t_i)$ œ t_i (e3)
t_i	g	U	B	B	B	B	G	B	G	B
	b	U	U	B	B	B	B	B	G	B
	u	U	U	U	B	B	B	B	B	B

Table 7. Equilibrium behavior chosen by agent in an open organization when $RBS_{BU}(\cdot)$ is large.

					$1 < A_1(b)$		$1 \$ A_1(b)$
		$1 < A_2(g)$ (a)	$1 < A_2(b)$ (b)	$1 < A_1(g)$ (c)	$1 < A_3(g)$ (d1)	$1 \$ A_3(g)$ (d2)	$1 < A_3(t_i)$ t_i 0 {g,b} (e)
t_i	g	U	B	B	B	G	B
	b	U	U	B	B	B	B

Table 8. Equilibrium behavior chosen by agent in a closed organization when $RBS_{BU}(\cdot)$ is large.

Suppose the Rate of Behavioral Substitution is sufficiently small. Tables 5 and 6 demonstrate that if $1 < A_1(g)$ (which is case (a) in the tables), that is, if the marginal benefit to agents of intrinsic type g (and hence of any worse type) of behaving as U rather than as G exceeds the marginal cost if sorted then all agent types behave as U regardless of the organization type.

If $A_1(g)$ # $1 < A_1(b)$ (which is case (b) in Tables 5 and 6), that is, if the marginal benefit to agents of type b (and hence u) of behaving as U rather than as G exceeds the marginal cost if sorted, and if the marginal benefit to agents of type g of behaving as U rather than as G does not exceed the marginal cost if sorted, then agent types b (and agent types u, if present) behave as U, while agent types g behave as G in both organization types. Conditions (2.8) (which state that the probabilities of being intrinsically good or bad in a closed organization are larger than the corresponding probabilities in an open organization) imply that more agents behave as good in a closed organization than in an open organization. Therefore, the closed organization is expected to make a larger outcome and to recoup less in penalties from the sorted agents.

If $A_1(b) \# 1 < A_1(u)$ (which is case (c) in Table 5), that is, if the marginal benefit to agents of type u for behaving as U rather than as G exceeds the marginal cost if sorted, and if the marginal benefit to agents of type b for behaving as U rather than as G does not exceed the marginal cost if sorted, then in equilibrium in an open organization u types behave as U, while g and b types behave as G. Note that there is no equilibrium in a closed organization when $A_1(b) \# 1$ regardless of z because, when there are no u types present, if all agents are sorted they behave as G, but the organization would not sort any agents if they behaved in this manner. For similar reasons, if $A_1(u) \# 1$, no equilibrium exists in either organization type regardless of z.

To conclude the case when the Rate of Behavioral Substitution is sufficiently small, we present our second set of equilibrium results comparing the two organization types:

Result 2. In equilibrium, when $z < r(U)$, $1 < A_1(g)$ and the Rate of Behavioral Substitution is sufficiently small, a closed organization is unambiguously inefficient. This is so because even though both organization types obtain the same outcome and recoup the same penalties, the closed organization expends resources screening agents before they are admitted to the organization.

Result 3. In equilibrium, when $z < E(r)$, $A_1(g) \# 1 < A_1(b)$ and the Rate of Behavioral Substitution is sufficiently small, a closed organization may or may not be inefficient. This is so because even though the closed organization expects a larger outcome than the open organization, it expends resources screening agents before they are admitted to the organization and also expects to recoup less in penalties.

Figure 1 illustrates these comparisons between organization types. The thick and solid horizontal lines represent E(r) values for a closed organization, while the thick and dashed lines are E(r) values for an open organization when the two organization types differ. Various sorting costs per agent are shown by thin and solid horizontal lines.

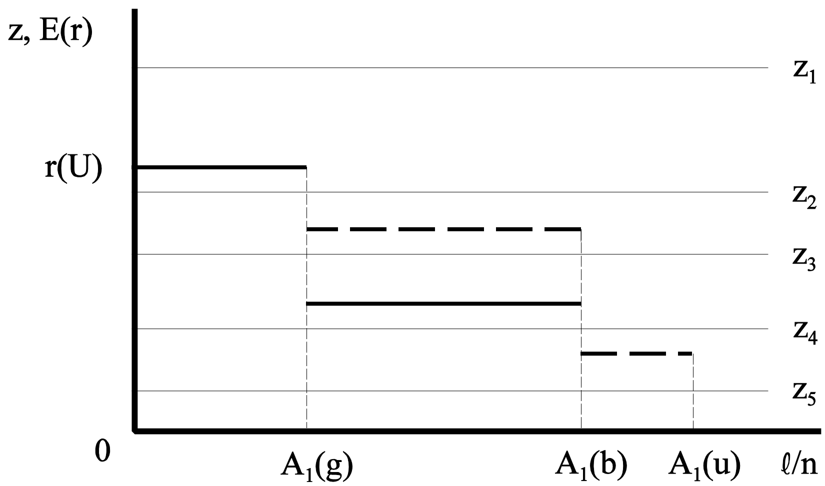

Figure 1

If z exceeds r(U), as z_1 does in Figure 1, then in equilibrium neither type of organization sorts any agents, and Result 1 applies. If the sorting cost is like z_2, then the only possible equilibrium for both open and closed organizations is one in which $\ell/n = 1 < A_1(g)$. That is, an equilibrium exists only if $A_1(g)$ happens to be larger than 1. This case corresponds to Result 2. The result follows because if $A_1(g) \leq 1$ then agents behave either as G and U, or all types behave as G. However, since $z > E(r)$ the organization will not sort any agents. Hence all agents will behave as U, and no equilibrium exists. This is in accord with the Corollaries to Propositions 3 and 5. The case when z is like z_3 is the same as the case of z_2 except that no equilibrium exists for the closed organization when $1 \geq A_1(g)$, and no equilibrium exists for both organization types if $1 \geq A_1(b)$. If z is like z_4, then the equilibrium depends on where 1 lies on the range from 0 up to (but not including) $A_1(b)$. If $1 < A_1(g)$, this case also corresponds to Result 2. If $A_1(g) \leq 1 < A_1(b)$, the implications are as in Result 3. If $1 \geq A_1(b)$, then no equilibrium exists in accord with the Corollaries to Propositions 3 and 5 again. If z is like z_5, the analysis is the same as the case of z_4 except that no equilibrium exists for the closed organization when $1 \geq A_1(b)$, and no equilibrium exists for both organization types when $1 \geq A_1(u)$.

Suppose now that the Rate of Behavioral Substitution is sufficiently large. By reasoning similarly to the analysis above we obtain the following results.[12]

Result 4. In equilibrium, when $z < E(r)$, and the Rate of Behavioral Substitution is sufficiently large, a closed organization is unambiguously inefficient when $1 < A_2(g)$, or when $A_2(u) \leq 1 < A_1(g)$ in an open organization and $A_2(b) \leq 1 < A_1(g)$ in a closed organization, or when $A_1(g) \leq 1 < A_1(b)$ and $1 < A_3(g)$, or when $A_1(b) \leq 1 < A_1(u)$ and $1 < A_3(t_i)$, for every $t_i \in \{g,b\}$, or when $1 \geq A_1(u)$ and $1 < A_3(t_i)$, for every $t_i \in \{g,b\}$. This is so because even though both organization types obtain the same outcome and recoup the same penalties, the closed organization expends resources screening agents before they are admitted to the organization.

Result 5. In equilibrium, when $z < E(r)$, and the Rate of Behavioral Substitution is sufficiently large, a closed organization may or may not be inefficient when $A_2(g) \leq 1 < A_2(b)$, or when $A_2(b) \leq 1 < A_2(u)$ in an open organization and $A_2(b) \leq 1 < A_1(g)$ in a closed organization, or when $A_1(g) \leq 1 < A_1(b)$ and $1 \geq A_3(g)$. This is so because even though the closed organization expects a larger outcome, it expends resources screening agents before they are admitted to the organization and also expects to recoup less in penalties than the open organization.

Results 1 - 5 can be used to formulate the following, arguably testable, implications regarding the choice of organizational type by any institutional entity.

(i) Organizations facing sufficiently high screening costs (because the screening cost per agent is high or because the frequency of intrinsically ugly types in the population is high) will choose the open type. This follows from all the results. If all agent types choose the same behavioral type regardless of organization type, Results 1, 2 and 4 imply that the open type is more efficient regardless of screening cost. If different agent types choose different behavioral types, Results 3 and 5 imply that open organizations are more efficient provided that screening costs are sufficiently high.

(ii) Organizations facing sufficiently high sorting costs relative to penalties recouped for agent misbehavior will choose the open type. This follows from Result 1.

[12] An appendix containing this analysis is available from the authors upon request. Also note that a figure analogous to Figure 1 could be drawn to illustrate these comparisons between organization types when the Rate of Behavioral Substitution is sufficiently large.

(iii) Organizations facing a sufficiently high frequency of intrinsically good types in the population or a sufficiently low frequency of intrinsically ugly types will choose the open type. This follows from all the results. If all agent types choose the same behavioral type regardless of organization type, Results 1, 2 and 4 imply that the open type is more efficient regardless of the frequency of good or ugly types. If different agent types choose different behavioral types, Results 3 and 5 imply that open organizations are more efficient provided the frequency of good (ugly) types is sufficiently high (low).

(iv) Organizations facing a sufficiently high outcome from agents who behave as good or bad relative to the outcome received from agents who behave as ugly will choose the closed organization type, provided that the sorting cost is sufficiently low. This follows from Results 3 and 5.

(v) Organizations that can recoup sufficiently high penalties for bad or ugly behavior will choose the open type, provided that the sorting cost is sufficiently low. If all agent types choose the same behavioral type regardless of organization type, Results 2 and 4 imply that the open type is more efficient regardless of penalties. If different agent types choose different behavioral types, Results 3 and 5 imply that open organizations are more efficient, provided the penalties are sufficiently high.

As mentioned above, the preceding results assume that the sorting cost per agent either exceeds the penalty expected to be received from any sorted agent in both open and closed organizations, or it does not exceed the expected penalty in both organizations. Assume now that both organization types face the same sorting cost per agent, but this cost exceeds the penalty expected to be received from any sorted agent in one organization type but not in the other type for the same range of possible R/n values. Then clearly this can occur only when the distribution of agent behavior differs across organization types. However, when the distribution of agent behavior differs, the expected penalty is always larger for the open organization. Therefore, if the sorting cost exceeds the expected penalty for one organization type only, it must be the closed type. This was discussed in passing in the preceding analysis of this section. Two examples are shown by lines z_3 (when $A_1(g) \# 1 < A_1(b)$) and z_5 (when $A_1(b) \# 1 < A_1(u)$) in Figure 1. The Corollary to Proposition 5 demonstrates that there is no equilibrium for the closed organization in this case. Hence we cannot compare the organizational types when this occurs.

In the preceding analysis, we also assumed that organizations of all types draw agents from the same pool. This assumption allowed us to infer that there are more intrinsically good and bad types in a closed organization than in an open organization (condition (2.8)). The implication is that closed organizations expect a larger outcome and less penalties to be recouped in the cases analyzed in Results 3 and 5. However, if the different types of organization draw agents from different pools, then we can never tell a priori how the distributions of intrinsic types differ in the two organization types, hence we cannot compare agent behavior (and hence expected outcome and penalties) in the two organizations at all. As an example, suppose that an open organization draws agents from a pool in which there is a plethora of intrinsically good types, and the closed organization draws agents from a pool in which good types are scarce. Then the likelihood of good types in a closed organization (even though it screens all agents *ex ante*) may be smaller than that of an open organization. One important implication of this observation for our analysis, and for any empirical research, is that we must be careful to determine whether agents are drawn from the same pool or not. If we are comparing organizational differences in, say, firms in the same industry and in the same geographical area, then it probably is a safe assumption that they are drawing agents from the same pool. However, if we are comparing US versus European or Japanese firms, they may be drawing agents from different pools, hence, comparing agent behavior may be impractical.

So far we have focused on efficiency from the organization's perspective. Here we extend the analysis to social efficiency. Clearly in cases where we cannot determine which organizational type is efficient from the organization's perspective, we cannot determine which type is socially efficient either. The cases analyzed in Results 3 and 5 fall into this category. In the remaining cases, that is, those presented in Results 1, 2 and 4, closed organizations are socially inefficient as well as being organizationally inefficient, only when all agent types behave as U. This is so because closed organizations deny entry to agents of type u who are replaced by g and b types. Since the u types would face no behavioral adjustment cost, while the g and b types do, closed organizations are less efficient both from the organization's perspective and from the agents' perspective. However, when all agent types behave as B in equilibrium, replacing u types with g and b types in closed organizations leads to lower behavioral adjustment costs, because the b types face no adjustment costs.[13] Hence, closed organizations may or may not be socially inefficient. Thus our next set of results is:

[13] Note that agents of type u face a "one-step" adjustment cost, similar to g type agents, while b type agents face no adjustment costs, which in this case is advantageous to a closed organization.

Result 6. In equilibrium, when $z \geq r(U)$, or when $z < r(U)$, $1 < A_1(g)$ and the Rate of Behavioral Substitution is sufficiently small, or when $z < r(U)$, $1 < A_2(g)$ and the Rate of Behavioral Substitution is sufficiently large, a closed organization is unambiguously socially inefficient. This is so because both organization types sort no agents and all agents behave as ugly, but the closed organization expends resources screening agents before they are admitted to the organization and the agents face higher behavioral adjustment costs.

Result 7. In equilibrium, when $z < E(r)$, $A_1(g) \leq 1 < A_1(b)$ and the Rate of Behavioral Substitution is sufficiently small, or when $z < E(r)$, the Rate of Behavioral Substitution is sufficiently large, and $A_2(u) \leq 1 < A_1(g)$ in an open organization and $A_2(b) \leq 1 < A_1(g)$ in a closed organization, or $A_1(g) \leq 1 < A_1(b)$ and $1 < A_3(g)$, or $A_1(b) \leq 1 < A_1(u)$ and $1 < A_3(t_i)$, for every $t_i \in \{g,b\}$, or $1 \geq A_1(u)$ and $1 < A_3(t_i)$, for every $t_i \in \{g,b\}$, or when $z < E(r)$, the Rate of Behavioral Substitution is sufficiently large, and $A_2(g) \leq 1 < A_2(b)$, or $A_2(b) \leq 1 < A_2(u)$ in an open organization and $A_2(b) \leq 1 < A_1(g)$ in a closed organization, or $A_1(g) \leq 1 < A_1(b)$ and $1 \geq A_3(g)$, a closed organization may or may not be socially inefficient. In some cases this is so because both organization types obtain the same outcome and recoup the same penalties, and agents in closed organizations face lower behavioral adjustment costs, but the closed organization expends resources screening agents before they are admitted to the organization. In other cases this is so because the closed organization expects a larger outcome than the open organization, and agents in the closed organization may face lower behavioral adjustment costs, but the closed organization expends resources screening agents before they are admitted to the organization and also expects to recoup less in penalties.

6. Extensions

The remaining analysis extends our results by relaxing some of our assumptions. Specifically, first we allow the average sorting cost to vary with the number of agents sorted; second we study the case when the organization has the power to precommit to a sorting frequency; and third we briefly consider sequential rather than simultaneous sorting.

6A. Economies and Diseconomies of Scale in Sorting Costs

The first extension we consider is average sorting costs that depend on the number of agents sorted. Even though we focused on the case of a constant average cost of sorting, z, our analysis applies much more generally. A constant average cost of sorting does imply that the organization will sort either all agents or none, but this would also be the case with economies of scale in the sorting cost. This is so because if it is worth sorting $0 < R < n$ agents (i.e., if the expected benefit per agent from collecting a penalty outweighs the sorting cost per agent when $0 < R < n$ agents are sorted), then it is worth sorting all n agents, which corresponds to all cases except (i) in Propositions 3 and 5. In addition, if it is not worth sorting all n agents, then it is not worth sorting any $0 < R < n$ agents. This corresponds to cases (i) in Propositions 3 and 5.

We now turn to the case in which diseconomies of scale are present. For ease of exposition, we assume there is a continuum of agents, and therefore the marginal sorting cost function is continuous in R/n. It can then be shown that an equilibrium always exists at the R/n such that the marginal cost of sorting equals the marginal benefit E(r).

The rationale for this result is as follows. Lemmas 2 and 4 show that for R/n values in different intervals agents will choose specific behavioral types, which lead to specific outcomes X that are observed by the organization. As shown previously, once the organization observes X it can infer exactly the number of agents behaving as each type and hence expects a unique penalty per sorted agent, E(r). If agents expect the organization to choose an R in a particular interval determined by the $A_1(\cdot)$ and $A_2(\cdot)$ values, such that the marginal cost of sorting equals the E(r) in that interval, then they behave in a way such that the X observed by the organization will lead the organization to expect the same E(r) as above. The organization will then choose the same R that agents expected. Note, however, that E(r) is a step function; hence, if the marginal sorting cost function crosses E(r) at a point where E(r) is discontinuous, no equilibrium exists. We demonstrate this by example in Figure 2, where we use the case of the closed organization when the Rate of Behavioral Substitution is small (see Table 3). Let the total sorting cost be $Z = z(R)R$. In Figure 2, the ZN curves are different marginal sorting cost functions, and $\Lambda(U)$ is the true frequency of agents behaving as U. E(r) can be r(U) (when all agents behave as ugly), or $\Lambda(U)r(U)$ (when only intrinsically bad types behave as ugly), or 0 (when all types behave as good).

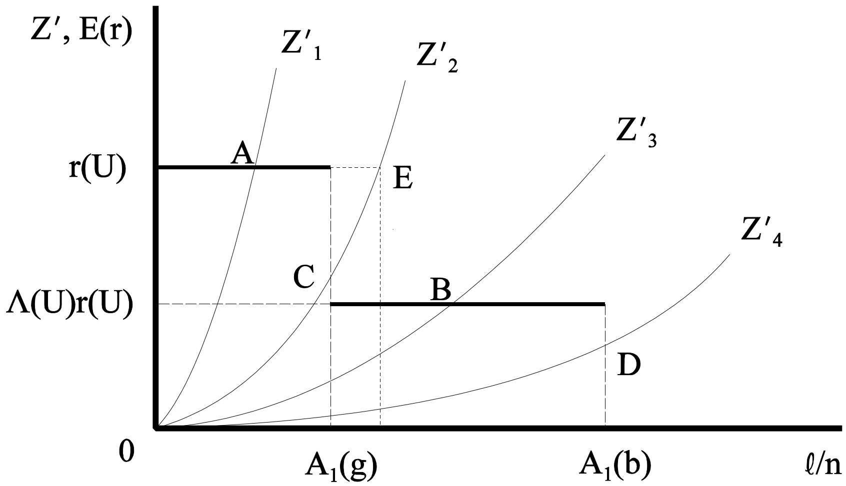

Figure 2

Clearly points A and B are equilibria in Figure 2. However, when ZN crosses the E(r) function where it is discontinuous, such as points C and D, there is no equilibrium. For instance, at C or any point to the right of C on ZN_2 there is no equilibrium because agents expect R/n $ $A_1(g) > 0$, and therefore the b types behave as U and the g types behave as G. But if the organization expects E(r) = Λ(U)r(U), it chooses R/n = 0 because ZN_2 at $A_1(g)$ exceeds E(r). At any point to the left of C on ZN_2, agents expect $A_1(g) >$ R/n $ 0, and hence all agents behave as U. But if the organization expects E(r) = r(U), it chooses the R/n corresponding to point E, where R/n > $A_1(g)$.[14] Thus there is no equilibrium when the marginal sorting cost is ZN_2.

As argued earlier, E(r) for the closed organization is always smaller or equal to that for the open organization, at the same R/n. Therefore closed organizations will never sort more agents than open organizations, if both organizations sort agents in equilibrium. This is shown by points A and B in Figure 3 where the solid line represents E(r) for the closed organization and the horizontal dashed line depicts E(r) for the open organization. The organizational efficiency implications are that closed organizations bear screening costs but less sorting costs, and expect to enjoy a larger outcome but

[14] It is straightforward to show that if there is no continuum of agents but, instead, the number of agents is discrete, then the equilibrium occurs at the largest R/n at which ZN # E(r). Note that if ZN > E(r) for R = 1, then the organization will not sort any agents in equilibrium.

recoup less in penalties. Further, similar to the constant sorting cost case, if Z' exceeds E(r) in one organization type, it must be the closed type. Then, no equilibrium exists for the closed organization, and hence no comparison of efficiency between organization types is possible. An example is shown by points C and D in Figure 3.

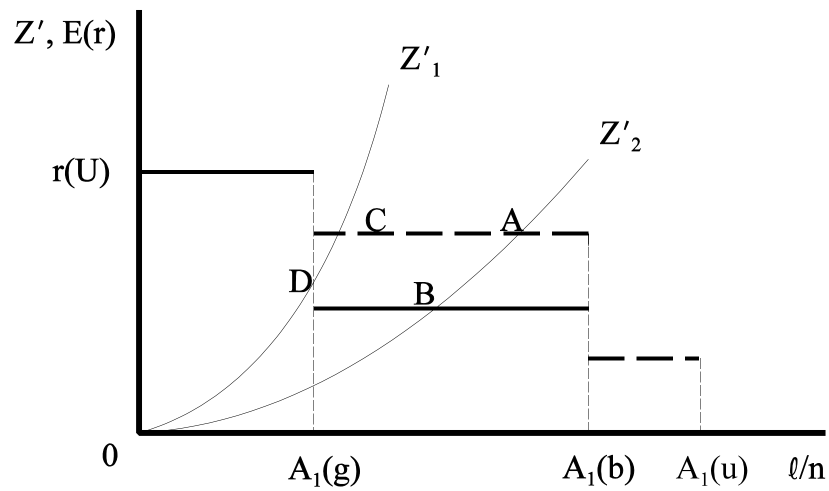

Figure 3

6B. Precommitment by the Organization

Next, we turn to the case when the organization has the power to precommit to a sorting frequency, assuming the cost per agent sorted is constant. The equilibrium of the game in this case is characterized by: (i) a strategy ρ for the organization determining the number of agents to be sorted; and (ii) a choice of behavioral type τ_i by each intrinsic type $t_i \in \{g,b,u\}$ of agent, $\tau_i(t_i|\rho) \in \{G,B,U\}$. The agents' strategies and the organization's strategy and beliefs must satisfy the following conditions. First, the organization chooses the strategy that maximizes its expected payoff shown in (6.1) for an open organization, or in (6.2) for a closed organization, given the agents' responses to the organization's choice:

(6.1) $$n \sum_{t_i} \left[p(t_i)\left[x_i(\tau_i(t_i|\rho)) + \frac{\rho}{n}\left[-z + r(\tau_i(t_i|\rho))\right]\right]\right],$$

$$(6.2) \qquad -E(m)s + n \sum_{t_i} \left[\mu(t_i)\left[x_i(\tau_i(t_i|R)) + \frac{R}{n}\left[-z + r(\tau_i(t_i|R))\right]\right]\right],$$

where $E(m) = n/[p(g) + p(b)]$ is the expected number of agents that have to be screened by the closed organization to get n good and bad agents. This is so because the number of good and bad agents obtained through screening follows a binomial distribution. Second, an agent's choice of behavioral type depending on his intrinsic type, and given the precommitted sorting frequency R/n, maximizes his expected payoff in (2.1).

Clearly, under precommitment, all equilibria in Propositions 3 and 5 where $z < E(r)$ and $R = n$ survive. This is so because if $z < E(r)$, then R should equal n even with precommitment to R. In these cases, precommitment has no value because it does not affect the equilibrium behavior of agents compared to the no-precommitment case. However, when $z \geq r(U)$, the equilibrium may differ from case (i) in Propositions 3 and 5 in which $R = 0$. With precommitment, the organization has to trade off the net cost of sorting (sorting cost minus penalties recouped) against the benefit from better agent behavior (higher output, X) and may sort $0 < R \leq n$ agents in equilibrium. Since the trade-off may or may not differ across organization types, they may or may not sort the same number of agents. The rationale for this is that even though closed organizations engage in *ex ante* screening, there may still be scope for extensive sorting to encourage good behavior. Precommitting to a low sorting frequency would invite opportunistic misbehavior by the agents. As a example we consider the small Rate of Behavioral Substitution case, where agents behave as either G or U in equilibrium, to show that, in fact, open organizations may sort the same or more agents than closed organizations. Refer to Figure 1 for the analysis below.

For ease of exposition, we assume there is a continuum of agents. We also assume the $A_1(\cdot)$ values are small in the sense that $1 > A_1(u)$ for generality (if they were large, the analysis would be simpler). Clearly, to minimize the loss from sorting, the organization will always sort the minimum number of agents necessary to induce the agent behavior it wants to implement. For example, if it is optimal for both organization types to implement good behavior by all agent types, then the open organization will sort exactly $A_1(u)$ agents, and the closed organization will sort exactly $A_1(b)$ agents. Thus an open organization chooses the sorting frequency among 0, $A_1(g)$, $A_1(b)$, or $A_1(u)$ that maximizes (6.1), while the closed organization chooses among 0, $A_1(g)$ or $A_1(b)$ to maximize (6.2). It is easy to see that precommitment now may have value in both organization types, because the equilibrium can occur at $R > 0$. For instance, if $x(G)$ is relatively large in the sense that $x(G) > x(U) + A_1(b)z$, then $R = 0$ will never be an equilibrium in either type of organization.

Finally, while no equilibrium exists in the no precommitment case when E(r) # z < r(U) as shown in the Corollaries to Propositions 3 and 5, there is always an equilibrium under precommitment in which the organization selects R to maximize (6.1) or (6.2), and the agents behave accordingly.

6C. Sequential Sorting

We now briefly show how the analysis can be extended when we allow for sequential instead of simultaneous sorting, keeping the cost per agent sorted constant. First note that if z $ E(r), and if an equilibrium exists, no sorting will occur in equilibrium even if sorting is sequential. Thus we focus on the case where z < E(r). We showed in the simultaneous sorting case that if an equilibrium with sorting exists the organization sorts all agents, R = n. We now characterize the optimal number of agents to be sorted when sorting is done sequentially. Specifically, we characterize the optimal stopping rule. For ease of exposition, we focus on the case where agents behave as either G or U in equilibrium. As argued in the preceding analysis, once the organization observes X, it can infer the actual number of agents behaving as G, nΛ(G), and as U, nΛ(U). Recall that, in this case, the organization can recoup a penalty from an agent only if it sorts that agent and finds him to be ugly. After R agents have been sorted and U(R) agents have been found to be ugly, the organization will not find it optimal to sort one more agent if

(6.3) $$\frac{n\Lambda(U) \,\&\, U(R)}{n \,\&\, R}\, r(U) \,\#\, z\,,$$

where [nΛ(U) - U(R)] / [n - R] is the probability that the next agent sorted is found to be ugly.[15] The actual R may be less than in the simultaneous sorting case if the organization finds a high proportion of the uglies early in the sorting process, in which case it is not worth sorting additional agents because the probability that the next agent sorted is found to be ugly is low.[16] In this setting, the agent's choice of behavioral type in equilibrium is best response to his expectation of R, and the organization's choice of R is best response to X in accord with (6.3). Comparing the equilibria in the two organization types and the associated organizational efficiency, when R is a random variable due to the sequential sorting, is the subject of future work. We refer the reader to the literature on stochastic games with stopping.

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[15] Note that the sequence U(1),U(2),...,U(R),... is a *submartingale* if E[U(R)] exists because with probability 1 E[U(R+1)] $ U(R).

[16] It is theoretically possible that R = n if n - 1 uglies were found in R - 1 sequential sortings. The organization then knows in advance that the last agent to be sorted will be found ugly, and it does sort him when z < r(U).

7. Conclusions

This paper develops a novel model of agent behavior in two stylized types of organization, open and closed, that differ in the degree to which each scrutinizes potential affiliates. An open organization does no screening of agents before they are admitted to the organization, while a closed organization screens all agents prior to admitting them. After observing the aggregate outcome, both organization types have the option to engage in *ex post* "auditing" of agent behavior (called sorting in our model) and penalize agents whose behavior is subpar. Agents can be of different intrinsic types (good, bad and ugly in our model) that differ in the degree to which they value misbehavior. Actual agent behavior (called behaviorally good, bad and ugly) depends on the short term net benefit of that behavior versus the expected penalty if caught misbehaving. The model is general enough to allow the organization to be a variety of institutions. For example, the organization could be a firm and the agents potential employees, or the organization could be a country and the agents potential immigrants, or a school or licensing authority dealing with applicants. The focus of the paper is agent behavior in these organization types and the associated efficiency implications.

One might expect *a priori* that closed organizations are more efficient than open organizations because one would anticipate better agent behavior and less equilibrium sorting, given that the closed organization screens agents and denies entry to the worst types. Surprisingly, this is not the case. Less sorting by the organization would invite opportunistic misbehavior by agents, and thus there would be a trade-off between payoffs to the organization and the costs of screening and sorting. We show that under quite general conditions regarding the sorting costs, in particular when the sorting costs per agent are constant or declining because of economies of scale, the closed organization will engage in the same amount of sorting as the open organization. Specifically, either all agents are sorted or none are sorted in equilibrium. Agents of the same intrinsic type, expecting the same amount of sorting, will behave identically in the two organization types. When agent behavior is uniform across intrinsic types, closed organizations turn out to be inefficient because they engage in costly *ex ante* screening without any improvement in agent behavior. If agent behavior differs across intrinsic types, then closed organizations end up with better agent behavior and thus may or may not be inefficient.

Not surprisingly, agent behavior is uniformly ugly if the organization does no screening. However, when agents are screened and all agent types behave uniformly, either all behave as ugly or all behave as bad. All behave as ugly when even the good intrinsic types find it "profitable" to misbehave because the short-term net benefit of behaving as ugly outweighs the penalty when sorted. Interestingly, agent behavior in equilibrium will never be uniformly good. This is so because if all agents behaved as good the organization would not want to sort any agents, in which case all agents would behave as ugly. A prerequisite for this result is that the organization cannot precommit to a sorting frequency.

When we extend the analysis to the precommitment case, we find that precommitment may or may not have value. Specifically, precommitment has no value when the sorting cost is less than the penalties expected to be recouped through sorting, because the organization will still find it optimal to sort all agents. By contrast, precommitment has value when there are equilibria under precommitment while no equilibria exist under no-precommitment, and in the case in which the sorting cost exceeds the penalties expected to be recouped, because instead of sorting no agents, the organization may precommit to sorting some agents at a loss in order to induce better behavior. We conjecture that closed organizations may sort less agents in equilibrium than open organizations, in order to induce the same agent behavior.

We also extend the analysis to allow for decreasing returns to scale in sorting, or for sequential rather than simultaneous sorting. In both cases we find that the number of sorted agents can be intermediate; that is, other than all or none. Under decreasing returns to scale we also find that closed organizations will never sort more agents in equilibrium than open organizations, and when the organizations sort a different number of agents, closed organizations induce better agent behavior even though they sort fewer agents.

We raised two main questions in the introduction: First, how does organization type affect agent behavior and organization payoff, and second, what factors determine the choice of organizational type? We answer the first question thoroughly as summarized above. Answers to the second question rely heavily on our analysis of the efficiency of different organization types. If all agent types behave as ugly or all as bad in equilibrium, then any organization will choose to be open to avoid the screening costs. If agent behavior is a mixture of types, then the choice of organization type depends on the trade-off between payoffs to the organization and the costs of screening and sorting.

Our main analysis yields the following implications. Organizations facing sufficiently high screening costs or, interestingly, sufficiently high sorting costs per agent relative to penalties recouped for agent misbehavior, will choose the open organization type. If the organization believes that the probability of ugly intrinsic types is sufficiently high or sufficiently low or the probability of good intrinsic types is sufficiently high, it will choose the open organization type again. Organizations facing a sufficiently high outcome from agents who behave as good or bad relative to the outcome received from agents who behave as ugly will choose the closed organization type, provided that the sorting cost is sufficiently low. Organizations that can recoup sufficiently high penalties for bad or ugly behavior will choose the open type, provided that the sorting cost is sufficiently low.

To conclude, our analysis shows that, overall, open organizations are more efficient than closed when all agent types behave uniformly across organization types. Open organizations are also socially efficient when all agents behave as ugly. However, when agent behavior is richer, either type of organization can be efficient under the right circumstances.

Appendix

Proof of Lemma 1. Given Assumptions 1b, 2 and 4, it can be shown that (3.8) and (3.11) hold. Given Assumptions 1a, 1b and 4, it follows that

$$(A1) \qquad A_1(g) < A_1(b) < A_1(u),$$

and

$$(A2) \qquad A_2(g) \# A_2(b) < A_2(u).$$

Suppose that $RBS_{BU}(t_i)$ satisfies condition (3.7). Given this condition and Assumptions 1a, 1b and 4, it follows that $A_1(u) < A_2(g)$. This finding, and conditions (A1) and (A2) above, complete the proof of (3.9). Note that (3.9) is consistent with condition (3.5), that is, given that (3.7) and (3.8) imply that $RBS_{BU}(t_i) < r(B) / r(U)$, œ t_i 0 {g,b,u}, it follows that $A_1(t_i) < A_2(t_i)$, œ t_i 0 {g,b,u}.

Suppose now that $RBS_{BU}(t_i)$ satisfies condition (3.10). Given this condition and Assumptions 1a, 1b, and 4, it follows that $A_2(u) < A_1(g)$. This finding, and conditions (A1) and (A2) above, complete the proof of (3.12). Note that (3.12) is consistent with condition (3.5), that is, given that (3.10) and (3.11) imply that $RBS_{BU}(t_i) > r(B) / r(U)$, œ t_i 0 {g,b,u}, it follows that $A_1(t_i) > A_2(t_i)$, œ t_i 0 {g,b,u}.

QED

Proof of Lemma 2. Assume that condition (3.7) holds. Condition (3.5) then implies that $A_1(t_i) < A_2(t_i)$ for every t_i. Starting from any intrinsic type t_i 0 {u,b,g}, the agent must decide whether to behave as U, B or G with the following expected payoffs:

$$(A3) \qquad E(U{*}t_i) = v(U) - k(U{*}t_i) - \frac{R}{n} r(U),$$

$$(A4) \qquad E(B{*}t_i) = v(B) - k(B{*}t_i) - \frac{R}{n} r(B),$$

$$(A5) \qquad E(G{*}t_i) = v(G) - k(G{*}t_i).$$

It can easily be verified that the agent will never behave as B, because $E(B{*}t_i) > E(U{*}t_i)$ and $E(B{*}t_i) > E(G{*}t_i)$ contradicts (3.5). Given Assumption 5, the agent will behave as U if

$$\text{(A6)} \qquad E(U{*}t_i) > E(B{*}t_i)$$

and

$$\text{(A7)} \qquad E(U{*}t_i) > E(G{*}t_i).$$

By rearranging terms, it can easily be shown that condition (A6) is equivalent to

$$\text{(A8)} \qquad \frac{R}{n} < A_2(t_i).$$

Similarly, condition (A7) is equivalent to

$$\text{(A9)} \qquad \frac{R}{n} < A_1(t_i).$$

However, since $A_1(t_i) < A_2(t_i)$, it follows that the agent will behave as U if (A9) is satisfied (in which case (A8) is automatically satisfied). This fully proves (3.13a) and partially proves (3.13b) and (3.13c).

Given Assumption 5, the agent will behave as G if

$$\text{(A10)} \qquad E(G{*}t_i) \ \$ \ E(B{*}t_i)$$

and

$$\text{(A11)} \qquad E(G{*}t_i) \ \$ \ E(U{*}t_i).$$

By rearranging terms, it can be shown that condition (A10) is equivalent to

$$\text{(A12)} \qquad \frac{R}{n} \ \$ \ A_1(t_i) \ \frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}.$$

Opposite to (A9), condition (A11) is equivalent to

$$\text{(A13)} \qquad \frac{R}{n} \ \$ \ A_1(t_i).$$

It follows that the agent will behave as G if (A13) is satisfied (in which case (A12) is automatically satisfied because (3.7) implies that $RBS_{BU}(t_i)$ is smaller than $r(B) / r(U)$). The findings above along with (3.9) in Lemma 1 complete the proof of (3.13b) and (3.13c) and prove (3.13d).

Assume that condition (3.10) holds. Condition (3.5) then implies that $A_1(t_i) > A_2(t_i)$ for every t_i. Starting from any intrinsic type $t_i \in \{u,b,g\}$, the agent must decide whether to behave as U, B or G with expected payoffs as in (A3), (A4) and (A5) above.

The agent will behave as U if (A6) and (A7) hold, which are equivalent to (A8) and (A9). However, since $A_1(t_i) > A_2(t_i)$, it follows that the agent will behave as U if (A8) is satisfied (in which case (A9) is automatically satisfied). This fully proves (3.14a) and partially proves (3.14b) and (3.14c).

Given Assumption 5, the agent will behave as B if

(A14) $$E(B|t_i) > E(G|t_i)$$

and

(A15) $$E(B|t_i) \geq E(U|t_i).$$

By rearranging terms, it can be shown that condition (A14) is equivalent to

(A16) $$\frac{R}{n} < A_1(t_i)\,\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}.$$

Condition (A15) is equivalent to

(A17) $$\frac{R}{n} \geq A_2(t_i).$$

It follows that the agent will behave as B if

(A18) $$A_2(t_i) \leq \frac{R}{n} < A_1(t_i)\,\frac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}}.$$

Note that (3.10) and (3.11) imply $\dfrac{RBS_{BU}(t_i)}{\frac{r(B)}{r(U)}} > 1, \forall t_i$. This along with (A18) and (3.12) in Lemma

1 complete the proof of (3.14b) and (3.14c), prove (3.14d), and partially prove (3.14e), (3.14f) and (3.14g).

Given Assumption 5, the agent will behave as G if (A10) and (A11) hold, which are equivalent to (A12) and (A13). It follows that the agent will behave as G if (A12) is satisfied (in which case (A13) is automatically satisfied because (3.10) implies that $RBS_{BU}(t_i) > r(B) / r(U)$). Conditions (A12) and (A18) along with (3.12) in Lemma 1 complete the proof of (3.14e), (3.14f) and (3.14g). QED

Proof of Proposition 3. First note that the organization's assessments of the frequencies of agents behaving as any of the behavioral types after observing aggregate outcome X , $\Phi(\tau_i{*}X) = \varphi(\tau_i{*}X)/n$, are calculated in accordance with the discussion following the characterization of the equilibrium at the end of section 2. Then the expected penalty E(r) is calculated in accordance with these assessments. Recall that penalties satisfy Assumption 4. Assume condition (3.7) holds.

(i) If $z \$ E(r)$, agents know the organization will never find it worthwhile to sort any agents. Given $R(\cdot) = 0$ and Assumptions 1, 2 and 3, all agent types behave as ugly; that is, $\tau_i(t_i) = U$, œi, œt_i.

(ii) If $z < E(r)$ and $A_1(g) > 1$, then at the largest possible $R(\cdot)$ (i.e., at $R(\cdot) = n$), it follows that $R(\cdot) = n < n A_1(g)$. Therefore, $R(\cdot) / n < A_1(g)$, œ$R(\cdot)$. Condition (3.13a) in Lemma 2 implies that $\tau_i(t_i) = U$, œi, œt_i. Given this behavior, and since $z < E(r)$, the organization will sort $R(\cdot) = n$ agents.

(iii) If $z < E(r)$, $A_1(g) \# 1$ and $A_1(b) > 1$, then it follows that $n A_1(g) \# R(\cdot) = n < n A_1(b)$. Condition (3.13b) in Lemma 2 implies that $\tau_i(g) = G$, œi, and $\tau_i(t_i) = U$, œi, œt_i.0 {b,u}. Given this behavior, and since $z < E(r)$, the organization will sort $R(\cdot) = n$ agents.

(iv) If $z < E(r)$, $A_1(b) \# 1$ and $A_1(u) > 1$, then it follows that $n A_1(b) \# R(\cdot) = n < n A_1(u)$. Condition (3.13c) in Lemma 2 implies that $\tau_i(t_i) = G$, œi, œt_i 0 {g,b}, and $\tau_i(u) = U$, œi. Given this behavior, and since $z < E(r)$, the organization will sort $R(\cdot) = n$ agents.

The proof of the case when (3.7) holds is completed by noting that no PBE exists when $A_1(u) \# 1$. This is so because when $A_1(u) \# 1$ there are two cases: either $R(\cdot)/n \$ A_1(u)$ or $R(\cdot)/n < A_1(u)$. In the former case, it follows from (3.13d) that $\tau_i(t_i) = G$, œi, œt_i. Given this behavior, $R(\cdot) = 0$. But if $R(\cdot) = 0$, then $\tau_i(t_i) = U$, œi, œt_i. Thus there is no PBE. In the latter case it follows that $R(\cdot) < n$. Further, $\tau_i(t_i)$ depends on how much smaller $R(\cdot)/n$ is relative to $A_1(u)$. The three possibilities are given by conditions (3.13a), (3.13b) and (3.13c) in Lemma 2. Given that $z < E(r)$, it is optimal for the organization to sort all agents; that is, $R(\cdot) = n$. Thus there is no PBE in this case either. To conclude, no PBE exists in pure strategies when $A_1(u) \# 1$.

Assume now that condition (3.10) holds. The proof of case (i) is identical to that when (3.7) holds and thus is omitted. The proof of cases (ii) - (v) is analogous to cases (ii) - (iv) when (3.7) holds except that conditions (3.14a) - (3.14d) in Lemma 2 are used rather than (3.13a) - (3.13c), so that the critical parameters are primarily the $A_2(t_i)$ values. Thus whereas the choice before was between U and G, now it is between U and B. The proof for cases (vi) - (xiii) uses conditions (3.14e) - (3.14g) in Lemma 2. Note that condition (3.16) is used in cases (viii) - (xiii). The proof is analogous to those above except that the critical parameters are now the $A_1(t_i)$ and $A_3(t_i)$ values (the latter depending on $RBS_{BU}(t_i)$); hence, the agents' behavioral choice is between B and G.

The proof of the case when (3.10) holds is completed by noting that no PBE exists when $A_1(u)$ # 1 and $A_3(t_i)$ # 1, œt_i. The reasoning is similar to that when (3.7) holds and $A_1(u)$ # 1. QED.

Proof of Corollary to Proposition 3. Given any putative equilibrium in which E(r) # z < r(U), it follows that R(X) = 0 regardless of X. But if agents expect R(·) = 0, then $\tau_i(t_i)$ = U œi, œt_i; that is, all agents behave as U. However, the condition that E(r) # z < r(U) rules out putative equilibria in which every agent behaves as U, because E(r) would equal r(U) in that case.[17] QED

[17] However, recall that there can be an equilibrium in which every agent behaves as U and R(X) = 0 if z $ r(U), which is case (i) in Proposition 3.

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