Optimal selling mechanisms with costly information acquisition

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November 19, 2001

This is an extremely preliminary version of the paper, only intended for seminar presentation at North Carolina State University on 26 November 2001. Although we believe that most of the results are correct, we cannot guarantee it. Please do not quote and certainly do not distribute without asking permission, but comments are most welcome.

1 Introduction

One of the basic assumptions in most of the mechanism design literature is that agents have private information about their type at the outset. The problem of a principal then is to design an optimal mechanism that induces the agents to reveal their private information. However there are many environments in which agents do not have full information in advance about their type and need to acquire this information at a cost. For instance, when the government sells spectrum to Telecommunication companies, the

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companies need to expand resources in order to find out how much they value this spectrum, especially if they are going to use the spectrum to offer new services for which the underlying technologies are not yet fully developed. Likewise, if a company auctions its existing assets to rivals (take for instance the recent sale of TWA assets to American Airlines), the potential buyers need to study the value of these assets and evaluate the potential synergies between these assets and their existing assets. In environments in which buyers need to expand resources in order to find out their valuations, the seller needs to take into account the costs of information acquisition and design a mechanism that, among other things, induces buyers to optimally acquire information.

In this paper we study the optimal design of selling mechanisms when the seller faces buyers who do not have private information at the outset. The seller then needs to decide what type of information to allow the buyers to acquire and to incite them to truthfully reveal their information when this could affect the acquisition of information by other buyers. The seller can also face the problem of inciting the buyers to expand the resources necessary to collect information.

In order to do so, we assume that a seller faces $N$ potential buyers. At the outset, each buyer has a cost of information acquisition, $c_i$ for buyer $i$, and a distribution of valuation $F_i$: if buyer $i$ spends $c_i$, he learns the value $v_i$ that he attaches to the object, which ex-ante has a distribution $F_i$. We assume that both $c_i$ and $F_i$ are common knowledge, but that $v_i$, once discovered in private information of bidder $i$.

In the first part of this paper, we show that in these circumstances, the asymmetry of information between buyers and sellers impose no cost on the seller: he obtains the same expected revenue than if he could observe the $v_i$s himself. Furthermore, we show that in special cases, the optimal mechanism takes very simple and intuitive forms. In section 5, we show that this result is not necessarily true if there is correlation between the types of the buyers. Hence, contrary to what happens in standard auction settings (Crémer and McLean (1985, 1988), correlation prevents full extraction of the surplus. All these results are obtained assuming that the seller can verify that a buyer actually acquires information when asked to do so. In section 6, we assume that the buyers have the possibility of faking the acquisition of information. In a special case of our general model, we show that our mechanisms can be redesigned to provide them with the proper incentives to acquire the information.

We believe that these results are noteworthy for two reasons. First, there is already a small literature that studies auctions when the auction format
influences the buyers' incentives to acquire information (e.g., McAfee and McMillan, 1987; Engelbrecht-Wiggans, 1993; Levin and Smith, 1994; Stegeman, 1996; Tan, 1992; Chakraborty and Kosmopoulou, 2001; Ye, 2001). This literature however assumes specific auction rules and derives the property of these rules, or, in some cases, the optimal mechanisms that satisfy these rules. As far as we know, no paper in the literature has studied the unconstrained optimal selling mechanism in this case. In the general case with independent types which is the case used studied in the rest of the literature, we find that the optimal mechanism can be implemented by a sequence of second price auctions, in which the potential sellers enter one by one. This might be a useful and practical technique in some cases. When it is not, we should identify the constraint that prevents the seller from using it, and solve for the optimal auction that satisfies these constraints.

We also believe that these results are noteworthy on more theoretical grounds. It is well know that, when agents and principal are risk neutral, adverse selection creates no loss of profits for the principal if the contract can be signed before the agents learn their type. This result has been proved for environments where the acquisition of information is exogenous. For the special case of auctions, we show that it also holds in some environments when the acquisition of information is itself an outcome of the mechanism. This paper suggests that more work is needed along these lines.

2 Finite number of ex ante i.i.d buyers

In order to build intuition for our results, we begin by considering a very special environment. The results described in this section will be considerably expanded in the sequel.

Consider an auction environment with a single object and \( N < \infty \) potential buyers, or briefly buyers \((N = 1, 2, 3, \ldots)\). The values of the object are independent and private across buyers. The value for buyer \( i \) \((\forall i = 1, \ldots, N)\) is an independent random draw from a commonly known distribution \( F \) with support \([0, \overline{v}]\). (Note that buyers can be ex ante heterogeneous.) The value of the object for the seller is normalized to zero. At the beginning, the realizations of the buyers' valuations are yet unknown. If a buyer \( i \) incurs a sunk cost \( c \), then he learns exactly his valuation \( v_i \) of the object and \( v_i \) becomes his private information. To ensure that buyers wish to participate in the mechanism, we assume that \( E[v] > c \). The time horizon consists of discrete periods, and all the buyers and the seller are assumed to have a common discounting factor \( \delta \in [0, 1] \). We assume that
only one buyer can acquire information in each period.

To simplify matters, we shall assume that each buyer must incur the cost \( c \) before he uses or consumes the object. In other words, we assume that a buyer cannot use or consume the object unless he spends a cost \( c \); in the process, the buyer learns his valuation of the object. This assumption rules out cases in which buyers can use or consume the object without knowing in advance how much they value it. In particular, this implies that the seller cannot pick a buyer at random and sell him the object at \( E[v] \) as the buyer would still have to incur a cost \( c \) before consuming the object. In what follows we shall therefore assume without a loss of generality that there is no loss (and there are generally gains) to have buyers incur the cost \( c \) and learn their valuations before they buy the object.

In addition, we assume that a buyer cannot acquire information unless he is granted access by the seller. This implies in turn that the seller has full control over whether and when each buyer learns his valuation, although the seller cannot observe that information. For instance, if the object is the rights to drill oil in a certain tract, then this assumption corresponds to the case where the seller can give the buyers geological information that allows them to assess the likelihood of finding oil. Without this geological information the buyers only know that the value of the oil in the ground is \( E[v] \). The assumption that the buyers’ valuations are private could be due to the fact that different firms have different costs for drilling oil. In this example, a buyer needs to incur a cost on geological studies (either before or after buying the right to drill oil) in order to decide how to optimally drill for oil. In the process of reviewing the geological information, the buyers also learns how much he values the oil in the ground. Alternatively, the seller might be the owner of some new technology. The buyers wish to acquire this technology but the seller can control their access to the blueprints. Without access to the blueprints, buyers only know the expected value of the technology but not its actual value to them. If the technology allows buyers to produce a new product then the valuations might be private, say because the buyers have different product lines in place so the new product may complement them to a different degrees. If the technology is cost-reducing then valuations might be private because the firms have different technologies in place. Again in this example, a buyer must eventually incur a cost in order to learn how to optimally utilize the new technology and in the process of doing so, the buyer learns his valuation of the technology. Or, it could be that the seller is a target firm and the buyers are potential acquires who need access to the target’s books in order to know how to utilize the target’s assets after the merger takes place. While doing that,
each acquirer also learns the post merger value of the merged firm. This value differs from one buyer to another because the merger creates different synergies between the target and the acquirer firms.

2.1 The optimal search

To study the first best case suppose that once a buyer learns his private valuation, the seller can also observe this valuation and make the buyer a take-it-or-leave-it offer. Suppose moreover that the seller can always go back to the highest valuation buyer from those that he already sampled. That is, suppose that the seller’s problem features perfect recall. Given these assumptions, the seller’s problem becomes completely equivalent to an optimal search mechanism with recall of the type that has been extensively studied in the literature. In particular, the first best case that we study in this subsection is a special case of Weitzman’s (1979) Pandora’s problem because we assume that buyers are ex ante identical: we study it from first principles without calling on Weitzman’s result in order to help build intuition.

In order to characterize the first-best solution, let us order the buyers in a sequence, 1, 2, ..., N. The seller can sample the buyers one at a time, learn the valuations of the sampled buyers and make a take-it-or-leave-it offer to the highest valuation buyer among those who were sampled. The state of the seller’s problem after the seller has already sampled m buyers is summarized by a pair \((v^*(m), N - m)\), where \(v^*(m)\) is the highest valuation among the m buyers that the seller has already sampled and \(N - m\) is the number of buyers that were not yet sampled. Suppose that the current state of the problem is \((v^*(N - 1), 1)\), so there is only 1 buyer left that was not sampled by the seller beforehand. Should the seller sample the last buyer as well? If he does, his discounted expected payoff, given that the current high valuation is \(v^*(N - 1)\) is

\[
S(v^*(N - 1)) = \delta \left[ \int_0^{v^*(N-1)} v^*(N-1) dF(v) + \int_{v^*(N-1)}^{v^*} v dF(v) \right] - c \tag{1}
\]

where the second equality follows from integration by parts. To understand this expression, note that when the valuation of the last buyer, \(v_N\), is such that \(v_N < v^*(N - 1)\), the seller will sell to the current highest valuation buyer at \(v^*(N - 1)\), and when \(v_N > v^*(N - 1)\), he will sell to the last
buyer at $v_N$. In any event, due to the delay associated with sampling the last buyer, the value of trade is discounted. If the seller does not sample the last buyer, he sells immediately to the buyer with the current highest valuation, so his payoff is $v^*(N - 1)$.

The seller will sample the last buyer if and only if $S(v^*(N - 1)) > v^*(N - 1)$, which is equivalent to

$$\Delta(v^*(N - 1)) \equiv \delta \left[ \overline{v} - \int_{v^*(N-1)}^{\overline{v}} F(v)dv \right] - v^*(N - 1) > c \quad (2)$$

The expression $\Delta(v^*(N - 1))$ is the discounted gross benefit from sampling the last buyer when the current highest valuation is $v^*(N - 1)$. The seller samples the last buyer if and only if the gross benefit from doing so exceeds the associated cost.

Since $\Delta'(x) = \delta F(x) - 1 < 0$ (the benefit from sampling the last buyer becomes smaller the higher is the current highest valuation), equation (2) implicitly defines a cutoff level, $k$, defined by $\Delta(k) = c$, or

$$\delta \left[ \overline{v} - \int_k^{\overline{v}} F(v)dv \right] = k = c, \quad (3)$$

such that when the state of the seller’s problem is $(v^*(N - 1), 1)$, the seller’s will sample the last buyer if and only if $v^*(N - 1) < k$. Otherwise, the seller is better-off not sampling the last buyer and instead, selling immediately to the current highest valuation buyer at $v^*(N - 1)$.

Now suppose that the seller has already sampled $N - 2$ buyers, so the current state of his problem is $(v^*(N - 2), 2)$. The seller can either sample one of the 2 remaining buyers (since buyers’ valuations are i.i.d., it does not matter which buyer is sampled), or can stop sampling and sell immediately to the buyer with the current highest valuation at $v^*(N - 2)$. If $v^*(N - 2) > k$, the seller will sample at most one more buyer. He will never sample both remaining buyers though, because after sampling buyer $N - 1$, the current high valuation will be $\max\{v^*(N - 2), v_{N-1}\}$; since $v^*(N - 2) > k$, the seller is better-off not sampling buyer $N$ as well (recall that we showed above that if there is only one more buyer left and the current high valuation exceeds $k$, the seller is better-off not sampling the last buyer). Hence the effective state of the problem is $(v^*(N - 2), 1)$ as there is at most one more buyer to sample. But earlier we saw that if $v^*(\cdot) > k$ and there is only one more buyer left to sample, the seller is better-off not sampling this last buyer. Since $v^*(N - 2) > k$, it follows that the seller is better-off not sampling buyer $N - 1$ as well and instead selling immediately at $v^*(N - 2)$.
Next suppose that \( v^*(N - 2) < k \). Should the seller stop and sell immediately at \( v^*(N - 2) \) or should he sample buyer \( N - 1 \)? To answer this question, note that earlier we found that the seller should sample another buyer if \( v^*(N - 1) < k \). Since \( v^*(N - 1) \geq v^*(N - 2) \) (note that by definition, \( v^*(N - 1) = \max\{v^*(N - 2), v_N\} \)), the inequality \( v^*(N - 1) < k \) implies that \( v^*(N - 2) < k \). Therefore the seller should sample at least buyer \( N - 1 \) (the option of also sampling buyer \( N \) makes the alternative of continuing to sample even more attractive).

We have therefore established that if \( v^*(N - 2) > k \), the seller should stop sampling and sell at \( v^*(N - 2) \), whereas if \( v^*(N - 2) < k \), he should continue to sample at least one more buyer. This implies that \( k \) is also the seller’s cutoff in state \((v^*(N - 2), 2)\): the seller should sample buyer \( N - 1 \) if and only if \( v^*(N - 2) < k \).

Continuing in the same way, we can establish that \( k \) is the seller’s cutoff in all states of the problem. In other words, at each state of the problem, the seller should compare the highest valuation at that point with \( k \) and continue to sample additional buyers if the current high valuation is below \( k \). As soon as the seller samples a buyer with a valuation that exceeds \( k \), he should make this buyer a take-it-or-leave-it offer equal to this buyer’s valuation.

### 2.2 Implementing the first-best solution

We now characterize a mechanism that allows the seller to replicate the first-best solution even if the seller can only control the buyer’s access to information but cannot observe this information along with the buyer. This mechanism consists of several stages. First, the seller orders the buyers in a sequence \( \{1, 2, ..., N\} \). Second, the seller charges each buyer \( i \) a participation fee \( T_i \), which will in general depend on the buyer’s position in the sequence. Third, the seller gives the first buyer in the sequence, buyer 1, access to information and once the buyer expands the cost \( c \) and becomes informed about his valuation, the seller makes him a take-it-or-leave-it offer, \( p_1 \). If buyer 1 accepts the offer, the seller sells the object to that buyer at \( p_1 \) and the mechanism stops. If buyer 1 rejects the offer, the seller turns to the second buyer in the sequence and gives him access to information. After buyer 2 becomes informed, the seller makes him a take-it-or-leave-it offer, \( p_2 \). If buyer 2 accepts the offer, the seller sells him at \( p_2 \) and the mechanism stops. Otherwise, the seller turns to the third buyer in the sequence and the process repeats itself. If all buyers in the sequence reject their respective offers, the seller holds a second price auction with all buyers as participants and sells the object to the highest bidder among them at a price equal to
the second highest bid.

We now show that this mechanism implements the first-best solution that we have characterized above. In order to replicate the first-best solution, the seller needs to design the sequence of prices, \( \{p_1, p_2, \ldots, p_N\} \), in such a way that each buyer in his turn will accept the seller’s offer only if the buyer’s valuation is above \( k \). The second price auction that is held if all \( N \) buyers reject their respective offers ensures that the buyer with the highest valuation among all buyers ends up buying the object. What we therefore need to show is that (i) it is possible to design the sequence of take-it-or-leave-it offers such that all buyers will accept their respective offers if and only if their valuations are above \( k \), and (ii) it is possible to design the participation fees such that the seller will extract the full surplus from each buyer.

We begin with the take-it-or-leave-it offers. Consider buyer \( i \) and suppose that this buyer has already paid the cost \( c \) and learned that his valuation is \( v_i \). If buyer \( i \) accepts the seller’s offer, \( p_i \), the buyer’s payoff gross of \( c \) and \( T_i \) is

\[
B^Y_i(v_i; p_i) = v_i - p_i. \tag{4}
\]

If buyer \( i \) rejects the seller’s offer, he still gets a chance to buy the object at the second price auction. According to the equilibrium hypothesis, the likelihood of reaching this stage is equal to the probability that all remaining \( N - i \) buyers will reject their respective offers. Since the seller designs the sequence \( \{p_1, p_2, \ldots, p_N\} \) such that each buyer accepts his respective offer if and only if the buyer’s valuation exceeds \( k \) (to replicate the first best solution), and since the distribution of buyers’ valuations is \( F(\cdot) \), the probability of reaching the second price auction is \( F(k)^{N-i} \). Now, let

\[
v^{N}_{-i} \equiv \max\{v_j | j \in \{1, \ldots, N\}; j \neq i\},
\]

be the highest valuation among all buyers but buyer \( i \). Since, the buyers’ valuations are \( i.i.d. \), the cumulative distribution of \( v^{N}_{-i} \) is given by \( F(\cdot)^{N-1} \). Therefore, buyer \( i \)’s discounted expected payoff from refusing to buy at price \( p_i \) and waiting for the eventual auction in the last round is

\[
B^N_i(v_i) = \delta^{N-i} F(k)^{N-i} \int_{0}^{v_i} \left( v_i - v^{N}_{-i} \right) dF(v^{N}_{-i} | v^{N}_{-i} < k),
\]

where \( F(v^{N}_{-i} | v^{N}_{-i} < k) \) is the cumulative distribution function of \( v^{N}_{-i} \), conditional on \( v^{N}_{-i} \) being less than \( k \) (the mechanism reaches the second price auction...
auction only if all valuations are less than \( k \). If \( v \leq k \), then

\[
F(v^N_{-i} \mid v_{-i}^N < k) = \frac{F(v_{-i}^N)^{N-1}}{F(k)^{N-1}}.
\]

As a result,

\[
B_i^N(v_i) = \delta^{N-i} F(k)^{N-1} \frac{\int_0^{v_i} (v_i - v_{-i}^N) dF(v_{-i}^N)^{N-1}}{F(k)^{N-1}}
= \delta^{N-i} F(k)^{N-1} \frac{\int_0^{v_i} F(v)^{N-1} dv}{F(k)^{N-1}},
\]

where the second equality follows from integration by parts.

The reserve price \( p_i \) is implicitly defined by the equation \( B_i^N(k) = B_Y^i(k; p_i) \). This equation has a unique solution as \( B_Y^i(k; 0) = k > B_i^N(k) > 0 = B_Y^i(k; k) \). Hence, the sequence \( \{p_1, p_2, \ldots, p_N\} \), ensures that the object is bought by the first buyer whose valuation exceeds \( k \), and if there is no such buyer, the object is bought by the buyer with the highest valuation among all \( N \) buyers. Notice that because \( B_i^N(k) \) is increasing in \( i \), the price \( p_i \) will be decreasing in \( i \). The first buyer will be offered a higher price than the last. Indeed, consider a buyer of type \( k \). If he waits for the second price auction, he is sure to win and the distribution of the payment that he has to make is independent of the rank he has been assigned. On the other hand, the probability that he will reach the auction stage is higher the greater his rank; therefore the greater the rank, the lower is the price \( p_i \) that will induce him to accept the offer.

It now remains to characterize the optimal participation fees, \( \{T^*_1, T^*_2, \ldots, T^*_N\} \).

To this end, recall that buyer \( i \) rejects the seller’s offer when \( v_i < k \), in which case his expected discounted payoff is \( B_i^N(v_i) \), and accepts it when \( v_i \geq k \), in which case his payoff is \( B_Y^i(v_i; p_i) \). Hence, buyer \( i \)'s discounted expected payoff from participation in the mechanism is

\[
\pi_i(v_i) = \int_0^k B_i^N(v_i) dF(v_i) + \int_k^\infty B_Y^i(v_i; p_i) dF(v_i) - c.
\]

Clearly, the seller would set \( T^*_i = \pi_i(v_i) \), for all \( i = 1, \ldots, N \), in order to extract the full surplus from each buyer. Buyers would agree to pay this amount as it leaves them with a 0 net expected payoff from participation (we assume that when indifferent, all buyers agree to participate, otherwise of course the seller can lower the participation fees slightly to make buyers strictly better-off if they participate).
3 A sequence of ex ante heterogenous buyers

Let us now consider an auction environment with a single object and $N < \infty$ potential buyers, or briefly buyers ($N = 1, 2, 3, \ldots$). The values of the object are independent and private across buyers. The value for buyer $i$ ($\forall i = 1, \ldots, N$) is an independent random draw from a commonly known distribution $F_i$, with support $[0, \bar{v}]$. (Note that buyers can be ex ante heterogeneous.) The value of the object for the seller is normalized to zero. At the beginning, the realizations of the buyers’ valuations are unknown. If a buyer $i$ incurs a sunk cost $c_i$, then he learns exactly his valuation $v_i$ of the object and $v_i$ becomes his private information. However, a buyer cannot acquire this information unless he is granted the access by the seller. We assume that the seller has full control over whether and when each buyer learns his valuation, although the seller cannot observe that information. The time horizon consists of discrete periods, and all the buyers and the seller are assumed to have a common discounting factor $\delta \in [0, 1]$. For tractability, we assume that only one buyer can acquire information in each period.

3.1 The first-best solution

The first-best solution, where the seller observes a buyer’s valuation along with the buyer during his information acquisition, corresponds to Weitzman’s (1979) Pandora’s problem. Here the seller needs only to conduct a costly search for the buyer with the highest valuation and then sell to this buyer at a price equal to the buyer’s valuation. If the seller samples buyer $i$ who was not sampled before, then his discounted expected payoff becomes

$$S_i(k) = \delta \left[ \int_0^{v^*(i-1)} kdF_i(v_i) + \int_{v^*(i-1)}^{\bar{v}} v_i dF_i(v_i) \right] - c_i. \quad (7)$$

The cutoff level $k_i$ could be interpreted as the reservation value of sampling buyer $i$. This reservation value depends only on the distribution of buyer $i$’s valuation and on the cost of sampling buyer $i$ but is independent of the distribution of other buyers’ valuations and of the cost of sampling other buyers. The seller simply compares the current highest valuation $k$ with $k_i$ and samples buyer $i$ if and only if $v^*(i - 1) < k_i$. Weitzman shows that the solution to the seller’s problem has the following two properties:

1. The buyers are ordered in a descending sequence of cutoff values so that $k_1 \geq \ldots \geq k_N$. If at any point the seller decides to sample another
buyer, he samples the buyer with the highest cutoff value among all
buyers who were not sampled up to that point.

2. The seller stops sampling whenever the current highest valuation ex-
ceeds the cutoff value of all buyers who were not yet sampled.

The two properties imply that the seller should begin by sampling the
buyer with the highest cutoff value provided that the cutoff value \( k_1 \) of this
buyer is positive (by not trading, the seller can get a payoff of 0). If the
valuation \( v_1 \) of buyer exceeds the cutoff value \( k_2 \) of buyer 2, the seller should
stop and sell to buyer 1. Otherwise, the seller should continue and sample
buyer 2, who has the second highest cutoff value. Now, the highest valuation
is \( v^*_2 = \max\{v_1, v_2\} \). If \( v^*_2 > k_3 \), the seller should stop sampling and sell to
either buyer 1 or to buyer 2, whoever has the highest valuation. Otherwise,
the seller should sample buyer 3. Repeat this process until the highest draw
among what have been sampled has exceeded the cutoff value for the next
round. If all buyers were already sampled, the seller sells to the buyer with
the highest valuation among all the \( N \) buyers.

3.2 Implementing the first-best solution

We shall construct a mechanism that allows the seller to replicate the first-
best solution even if the seller can only control a buyer’s access to informa-
tion but cannot observe this information along with the buyer. To avoid
triviality, let us assume that \( k_N > 0 \), so that it is worthwhile to include all
the buyers in the waiting list of information acquisition.

3.2.1 The Mechanism

The mechanism consists of several stages.

1. The seller labels the potential buyers as 1, \ldots, N in descending order
   of the cutoff values, so that \( k_1 \geq \ldots \geq k_N \).

2. The seller makes a take it or leave it offer to the buyers: you are
   allowed to participate in the mechanism if you pay the sum \( T_1 \).

3. The seller gives buyer 1 access to information. (As the seller is as-
   sumed to fully control a buyer’s access to information, buyer 1 at this
   point must acquire information about his own valuation.) Once the
   buyer expands the cost \( c_1 \) and becomes privately informed about his
   valuation \( v_1 \), the seller makes buyer 1 a take-it-or-leave offer at price
which will be defined below. If buyer 1 accepts the offer, the seller sells the object to that buyer at $p_1^1$ and the game stops. (It will be convenient to view this stage as a second-price auction with a single bidder (buyer 1) and a minimum acceptable bid (reserve price) $p_1^1$.)

4. If buyer 1 rejects the offer, the seller turns to buyer 2 and gives him access to information. After buyer 2 becomes privately informed about $v_2$, the seller holds a second-price auction with round- and buyer-specific reserve prices, $p_1^2$ for buyer 1 and $p_2^2$ for buyer 2, which will be defined below. In the sequel, a buyer’s bid is said to be admissible if it is greater than or equal to the reserve price assigned to the buyer. If at least one buyer submits an admissible bid, then the object is sold and the game ends. Otherwise, the seller turns to buyer 3, and the process continues.

5. If the process reaches a round where exactly $n$ buyers have acquired information and the object has not been sold, with $n < N$, then the seller holds a second-price auction with round- and buyer-specific reserve prices so that buyer $i$ is assigned a reserve price $p_n^i$. (The bidder who submits the highest admissible bid wins and pays either his reserve price or the highest losing admissible bid, whichever is higher.)

6. If the process reaches the last buyer in the sequence, then after this buyer learns his own valuation, the seller holds a second-price auction without reserve prices and sells the object to the highest bidder at price equal to the second high bid.

We can prove the following proposition.

**Proposition 1** There exists a transfers $T_i$, $i = 1, \ldots, N$ and sequence of prices $p_i^j$, $i \leq j$ such that the mechanism defined above implements the first best solution.

The proof computes the prices such that at round $j$ buyer $i$ such that $v_i = k_{j+1}$ is exactly equivalent between buying the object at price $p_i^j$ and going to the next round of play. Then, given that at each stage the mechanism defines a second price auction, it is easy to show that revealing once true type is an equilibrium strategy. (Although this is easily stated, the proof requires quite a bit more work.)
4 Implementation in the general independent case

In this section we consider a more general environment than the one considered earlier. Again, the seller has one unit to sell and searches for a buyer for the object. There are discrete time periods, \( t = 0, \ldots, T \), and a set \( I \) of potential buyers. The valuation of buyer \( i \in I \) (or buyer \( i \)'s type) is an independent random variable drawn from a commonly known distribution \( F_i \) with support \([0, v]\). The cost that buyer \( i \) must incur in order to learn his type is \( c_i \) and the buyer’s discount factor at period \( t \) is \( \delta_t \in [0, 1] \).

The environment that we will consider in this section differs from the one we studied earlier in two ways. First, we shall assume that in any given period, any number of buyers (instead of only one) can spend the cost \( c_i \) of learning their types. As before, we shall continue to assume that access to information can be completely controlled by the seller. Second, we shall also assume that when a buyer spends the cost \( c_i \), he does not necessarily learn his type immediately; rather a time \( \tau_i \geq 0 \) needs to pass from the period in which \( c_i \) is spent until buyer \( i \) can observe his type. That is, if buyer \( i \) spends the cost \( c_i \) in period \( t \), then he learns his type in period \( t + \tau_i \). This assumption is meant to capture the idea that in many real-life situations, e.g., a merger between two large firms or the sale of very complex systems, figuring out one’s valuation may be a complicated task and it may take considerable time before a buyer can fully study and digest the information provided by the seller.\(^1\) Furthermore, it would be possible to add other constraints on the problem (for instance a limit on the total number of buyers who can be “active” at any time), or use more general cost functions.

The aim of the exercise is to show that the seller can find a selling mechanism such that he will have the same surplus than if he had directly access to the information about the valuation of the sellers. In this case, the mechanism will not, in general, be a succession of auctions.

The assumption that learning one’s type may take time implies that, so long as the object is still unsold, in every period \( t \) the seller has three options: terminate the search and sell the object to buyer with the highest known

\(^1\)In addition there may be bureaucratic hurdles that the buyer must clear, like getting the approval of an antitrust authorities for a merger or getting the approval of the FDA for marketing a new drug or a food product. These hurdles may take considerable time to clear; until they are, the buyer may not know his valuation since bureaucratic agencies may impose certain restrictions on the deal (e.g., antitrust authorities often require the divestiture of some of the merged entity assets which obviously affect greatly the buyer’s valuation).
valuation, select a new set of buyers, \( I_t \), who will be allowed to spend the cost of learning their respective types, or wait. The last decision is equivalent to selecting \( I_t = \emptyset \), i.e., neither terminating the search nor allowing new buyers to learn their types. This decision may be optimal because at time \( t \) there may be buyers who already spent the cost of learning their types but still do not know it.

We begin by considering the no asymmetry of information case where the seller learns the buyer’s type along with the buyer. The aim of this exercise is to show lemma 2 which states that a measure of the discounted probability that an agent obtains the object is increasing in his valuation. This will imply in turn that we can implement the optimal search, as stated in theorem 2. Since the seller can extract the entire buyer’s valuation via a take-it-or-leave-it offer, this case is equivalent to a parallel search problem (see Wishwanath, 1992) in which the seller is looking for the highest valuation buyer. But, since here there is possibly a time gap between spending the search cost and learning a buyer’s type, the current problem is in general considerably harder than the one studied by Wishwanath. Nonetheless, we will show that whatever the solution to the problem is, we can construct an incentive compatible selling mechanism that will implement this solution.

Without asymmetry of information, the seller is interested to find the buyer with the highest valuation. Informally, the state \( S_t \) of buyer \( i \) at the end of period \( t \) can be of three different types:

1. He could not yet have been called upon to learn his valuation;
2. He could already have learned this valuation;
3. He could have already spent the amount \( c_i \) and know that he will learn his valuation in \( \theta_i \) periods.

The state of the system \( S_t \) is a list of state for every buyer. A strategy for the seller is a function from \( (S_t, t) \) into a set of actions at time \( t \): these actions can be either to stop the search, to wait, or to open some number of unopened boxes. The final version of the paper will have a proof of the following lemma, which for the purpose of this version we will assume is true.

**Lemma 1** There exists an optimal strategy such that the move of the seller at time \( t \) depends on the \( v_i \) for the buyers who have already been opened only through their maximum.
Now, fix the time $t$ and the state, except for the maximum type of the buyers whose $v_i$ is already known, $v^t_{\text{max}}$. Assume that for all periods $s = t, t + 1, \ldots, T$, the seller acts as if $v^t_{\text{max}}$ had obtained. Then the probability that the search stops at time $s \geq t$ is $P(s; v^t_{\text{max}})$ and the distribution probability of the highest new information obtained if the search stops in period $s$ is $F(v; s, v^t_{\text{max}})$. Because there is no reason to use deterministic strategies, we can restrict ourselves to search procedures where $P(s; v^t_{\text{max}})$ can only take the values 0 or 1. For the case where $P(s; v^t_{\text{max}}) = 1$, we write $F(v; s, v^t_{\text{max}}) = 0$ if $v < v^t_{\text{max}}$ and $F(v; s, v^t_{\text{max}}) = 1$ if $v \geq v^t_{\text{max}}$. (Note that the notations suppresses the dependance of the functions $P$ and $F$ on the state, except for $v^t_{\text{max}}$, and the period — this will create no ambiguity in what follows.) There will be periods such that $P(s; v^t_{\text{max}}) = 0$; for those $F$ can be defined arbitrarily, as long as it has a finite mean.

Now, assume that after having observed $v^t_{\text{max}}$ the seller hires an agent (not a buyer, some external agent) and tells him: "follow the optimal strategy for $\hat{v}^t_{\text{max}}$, and when the strategy tells you to stop tell me the maximum $v$ that you have found. At that point, I will choose either $v^t_{\text{max}}$ or the maximum $v$ that you have found." In this case, the expected revenue of the seller (discounted back to time 0) is

$$
\sum_{s=t}^{T} \delta_s P(s; \hat{v}^t_{\text{max}}) \left[ \int_0^{v^t_{\text{max}}} v^t_{\text{max}} dF(v; s, \hat{v}^t_{\text{max}}) + \int_{v^t_{\text{max}}}^\bar{v} v dF(v; s, \hat{v}^t_{\text{max}}) \right]
= \sum_{s=t}^{T} \delta_s P(s; \hat{v}^t_{\text{max}}) [v^t_{\text{max}} F(v^t_{\text{max}}; s, \hat{v}^t_{\text{max}}) + \bar{v} - v^t_{\text{max}} F(v^t_{\text{max}}; s, \hat{v}^t_{\text{max}}) - \int_{v^t_{\text{max}}}^{\bar{v}} F(v; s, \hat{v}^t_{\text{max}}) dv]
= \sum_{s=t}^{T} \delta_s P(s; \hat{v}^t_{\text{max}}) \left[ \bar{v} - \int_{v^t_{\text{max}}}^{\bar{v}} F(v; s, \hat{v}^t_{\text{max}}) dv \right].
$$

The seller is better off following the true optimal search strategy. Let $c(\hat{v}^t_{\text{max}})$ be the expected sum of the future costs $c_i$, again discounted to time 0, when following the search strategy corresponding to $\hat{v}^t_{\text{max}}$. The difference between expected revenue and expected cost is maximized for
\[ \dot{v}_\text{max} = v_\text{max}. \]  This implies

\[
\sum_{s=t}^{T} \delta_s P(s; v^t_\text{max}) \left[ \bar{v} - \int_{v^t_\text{max}}^{\bar{v}} F(v; s, v^t_\text{max}) \, dv \right] - c(v^t_\text{max}) \geq \sum_{s=t}^{T} \delta_s P(s; \dot{v}_\text{max}^t) \left[ \bar{v} - \int_{\dot{v}_\text{max}^t}^{\bar{v}} F(v; s, \dot{v}_\text{max}^t) \, dv \right] - c(\dot{v}_\text{max}^t).
\]

Similarly, interverting \( v^t_\text{max} \) and \( \dot{v}_\text{max}^t \) we have

\[
\sum_{s=t}^{T} \delta_s P(s; \dot{v}_\text{max}^t) \left[ \bar{v} - \int_{\dot{v}_\text{max}^t}^{\bar{v}} F(v; s, \dot{v}_\text{max}^t) \, dv \right] - c(\dot{v}_\text{max}^t) \geq \sum_{s=t}^{T} \delta_s P(s; v^t_\text{max}) \left[ \bar{v} - \int_{v^t_\text{max}}^{\bar{v}} F(v; s, v^t_\text{max}) \, dv \right] - c(v^t_\text{max}).
\]

For concreteness and without loss of generality, assume \( \dot{v}_\text{max}^t < v^t_\text{max}. \) Adding these two inequalities we obtain

\[
\sum_{s=t}^{T} \delta_s P(s; v^t_\text{max}) \int_{v^t_\text{max}}^{\dot{v}_\text{max}^t} F(v; s, v^t_\text{max}) \, dv \\
\geq \sum_{s=t}^{T} \delta_s P(s; \dot{v}_\text{max}^t) \int_{\dot{v}_\text{max}^t}^{v^t_\text{max}} F(v; s, \dot{v}_\text{max}^t) \, dv.
\]

(8)

Now, given that the function \( F(v; s, v^t_\text{max}) \) is increasing in \( v \) we have

\[
\sum_{s=t}^{T} \delta_s P(s; v^t_\text{max}) F(v; s, v^t_\text{max}) \leq \sum_{s=t}^{T} \delta_s P(s; v^t_\text{max}) F(v^t_\text{max}; s, v^t_\text{max})
\]

for all \( v \in [\dot{v}_\text{max}^t, v^t_\text{max}] \)

and therefore

\[
\sum_{s=t}^{T} \delta_s P(s; v^t_\text{max}) \int_{v^t_\text{max}}^{\dot{v}_\text{max}^t} F(v; s, v^t_\text{max}) \, dv \\
\leq \left[ \sum_{s=t}^{T} \delta_s P(s; v^t_\text{max}) F(v^t_\text{max}; s, v^t_\text{max}) \right] (v^t_\text{max} - \dot{v}_\text{max}^t).
\]

(9)
Similarly, we have
\[
\sum_{s=t}^T \delta_s P(s; \hat{v}_t) F(v; s, \hat{v}_t) \geq \sum_{s=t}^T \delta_s P(s; \hat{v}_t) F(\hat{v}_t; s, \hat{v}_t)
\]
for all \( v \in [\hat{v}_t, \hat{v}_t] \), and therefore
\[
\sum_{s=t}^T \delta_s P(s; \hat{v}_t) \int_{\hat{v}_t}^{v_{max}} F(v; s, \hat{v}_t) dv
\geq \left[ \sum_{s=t}^T \delta_s P(s; \hat{v}_t) F(\hat{v}_t; s, \hat{v}_t) \right] (v_{max} - \hat{v}_t). \quad (10)
\]

Inequalities (8), (9) and (10) prove the following lemma.

**Lemma 2** In an optimal search strategy, the function
\[
\sum_{s=t}^T \delta_s P(s; v_t) F(v_t; s, v_{max})
\]
is increasing in \( v_{max} \).

Assume now that the seller follows the optimal search strategy and consider a situation where we know the state at period \( t-1 \), and therefore \( v_{max} \), and the valuation \( v_i \) of buyer \( i \) whose valuation becomes known in period \( t \). Let \( k = 1, \ldots, \hat{k} \) be the buyers other than \( v \) who are also observing their types at period \( t \). Define (with \( v_{max}^0 = 0 \))
\[
q_i(v_i) = \begin{cases} 
0 & \text{if } v_i \leq v_{max}^{t-1} \\
\prod_{k=1}^{\hat{k}} F_k(v_i) \times \left[ \sum_{s=t}^T \delta_s P(s; v_i) F(v_i; s, v_i) \right] & \text{if } v_i > v_{max}^{t-1}.
\end{cases}
\]
It is straightforward that \( q_i \) is increasing in \( v_i \).

We now turn to the asymmetric information case. Invoking the revelation principle, we can restrict attention without a loss of generality to incentive compatible direct revelation mechanisms in which buyer \( i \) spends a cost \( c_i \) when invited by the seller and after he learns his type makes a (truthful) report to the seller, \( v_i \). Given a report \( v_i \), the seller requires the buyer to pay \( \rho_i(v_i) \) and acts as if \( v_i \) were the true type of the buyer (i.e., if \( v_i \) is the maximum announcement, he follows the search procedure corresponding to \( v_{max}^t = v_i \) and gives the object to agent \( i \) if this search procedure does not yield any other buyer who announce a higher type).
Proposition 2 There exists an incentive compatible mechanism that implements the optimal search strategy.

Proof. Since the seller can charge each buyer a fee before giving him access to information, it is obvious that he can captures the entire expected rent generated by the mechanism. Hence, we only need to show that we can find a \( \rho_i \) such that buyer \( i \) is induced to tell the truth.

If an agent announces that he is of type \( \hat{v}_i \) when he is really of type \( v_i \), his expected utility will be \( q_i(\hat{v}_i)v_i \). Hence, incentive compatibility requires

\[
q_i(v_i)v_i - \rho_i(v_i) \geq q_i(\hat{v}_i)v_i - \rho_i(\hat{v}_i), \quad \forall v_i, \hat{v}_i \in [0, \bar{v}],
\]

and

\[
q_i(\hat{v}_i)v_i - \rho_i(\hat{v}_i) \geq q_i(v_i)\hat{v}_i - \rho_i(v_i), \quad \forall v_i, \hat{v}_i \in [0, \bar{v}].
\]

Adding (11) and (12) and rearranging terms,

\[
v_i [q_i(v_i) - q_i(\hat{v}_i)] \geq \rho_i(v_i) - \rho_i(\hat{v}_i) \geq \hat{v}_i [q_i(v_i) - q_i(\hat{v}_i)].
\]

Assuming without a loss of generality that \( v_i > \hat{v}_i \) and taking the limit of (13) as \( \hat{v}_i \to v_i \), yields

\[
\rho'_i(v_i) = q'_i(v_i)v_i,
\]

almost everywhere. Integrating (14), we get

\[
\rho_i(v_i) = \int_0^{v_i} q'_i(z)zdz, \quad \forall v_i \in [0, \bar{v}].
\]

To complete the proof, we need to show that payments as in (15) induce truth telling. Since Lemma \( x \) ensures that \( q_i(\cdot) \) is increasing, it follows from (15) that

\[
\rho_i(v_i) - \rho_i(\hat{v}_i) = \int_{\hat{v}_i}^{v_i} q'_i(z)zdz \\
\geq \int_{\hat{v}_i}^{v_i} q'_i(z)v_i dz \\
= v_i [q_i(v_i) - q_i(\hat{v}_i)].
\]

This inequality implies that (11) is satisfied. \( \blacksquare \)

The proposition shows that whatever the optimal search strategy is, we can construct an incentive compatible direct revelation mechanism that implements it. Therefore, the seller can completely overcome the informational asymmetry problem.
5 Buyers with correlated valuations

In this section we show that when buyers’ valuations are correlated, it may no longer be possible to design an incentive compatible mechanism that fully extracts the surplus. To this end consider the following simple example: there are two potential buyers. The cost that each buyer incurs when learning his/her type is $c$, where $1 < c < 2$. The buyers’ valuations, $v_1$ and $v_2$, can take 3 possible values: 1, 3, and 10. The joint probability distribution over $v_1$ and $v_2$ is indicated in the following table:

<table>
<thead>
<tr>
<th>$v_1$</th>
<th>1</th>
<th>3</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.28</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>10</td>
<td>0.01</td>
<td>0.16</td>
<td>0.18</td>
</tr>
</tbody>
</table>

For instance, $\Pr(v_1 = 3, v_2 = 10) = 0.16$. Notice that the probability distribution is not degenerate, and that there is positive correlation between the buyers’ valuations (the diagonal terms are larger than the off-diagonal terms).

Before proceeding, it should be noted that if the buyers knew their valuations at the outset, the seller would have been able to design a dominant strategy selling mechanism that fully extracts the surplus. This is because the joint probability distribution over $v_1$ and $v_2$ satisfies the Crémer-McLean (1985, 1988) full-rank condition. To see that, note that in this example, the matrix of conditional probabilities whose rows are parametrized by $v_i$ and whose columns are parametrized by $v_j$ is given by

$$Q = \begin{pmatrix}
\frac{28}{30} & \frac{1}{35} & \frac{1}{35} \\
\frac{1}{30} & \frac{18}{35} & \frac{16}{35} \\
\frac{1}{30} & \frac{16}{35} & \frac{18}{35}
\end{pmatrix}.$$

The first column in $Q$ describes the probability distribution of $v_i$ conditional on $v_j = 1$, the second column describes the probability distribution of $v_i$ conditional on $v_j = 3$, and third column describes the probability distribution of $v_i$ conditional on $v_j = 10$. For instance, the probability that $v_i = 1$ conditional on $v_j = 3$ is $1/35$. The Crémer-McLean (1985, 1988) full-rank condition states that there exists a dominant strategy selling mechanism that fully extracts the surplus if $Q$ has full rank. To check that $Q$ has a full rank, we need to show that there does not exist a vector $\rho = \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}$
where \( \rho \neq 0 \), such that \( Q \times \rho = 0 \). This equality can also be written as,
\[
\begin{align*}
\frac{28}{30} \rho_1 + \frac{1}{35} \rho_2 + \frac{1}{35} \rho_3 &= 0, \\
\frac{1}{30} \rho_1 + \frac{18}{35} \rho_2 + \frac{16}{35} \rho_3 &= 0, \\
\frac{1}{30} \rho_1 + \frac{16}{35} \rho_2 + \frac{18}{35} \rho_3 &= 0.
\end{align*}
\]
Subtracting the third equation from the second equation reveals that \( \rho_3 = \rho_2 \). Substituting \( \rho_3 = \rho_2 \) in the first and second equations yields
\[
\begin{align*}
\frac{28}{30} \rho_1 + \frac{2}{35} \rho_2 &= 0, \\
\frac{1}{30} \rho_1 + \frac{34}{35} \rho_2 &= 0.
\end{align*}
\]
These equations imply in turn that \( \rho_1 = \rho_2 = \rho_3 = 0 \). Hence, \( Q \) has full-rank.

### 5.1 Optimal search

Let us now turn to the optimal search procedure under full information. Under full information, the seller invites one buyer, pays the cost \( c \) of learning the buyer’s valuation and then either makes this buyer a take-it-or-leave-it offer or invites the second buyer. In the latter case, the seller again pays a cost \( c \), learns the second buyer’s valuation and then makes a take-it-or-leave-it offer to the buyer with the highest valuation between the two. Assume without a loss of generality that buyer 1 is invited first. If \( v_1 = 10 \), the seller sells to buyer 1 at a price of 10 and his net payoff is \( 10 - c \). If \( v_1 = 1 \), the seller can either sell to buyer 1 at a price of 1 and make a net payoff of \( 1 - c \) or invite buyer 2, in which case his net expected payoff (using the probabilities in the table above) is
\[
1 \times \frac{28}{30} + 3 \times \frac{1}{30} + 10 \times \frac{1}{30} - 2c = 1.366 - 2c.
\]
Since by assumption, \( c > 1 \), \( 1 - c > 1.366 - 2c \), implying that the seller is better-off selling to buyer 1 at a price of 1. If \( v_1 = 3 \), the seller can either sell to buyer 1 at a price of 3 and make a net payoff of \( 3 - c \), or invite buyer 2. If the seller invites buyer 2, then he will eventually sell to buyer 1 if \( v_2 = 1 \), sell to buyer 2 if \( v_2 = 10 \), or sell to either buyer 1 or 2 if \( v_2 = 3 \). Hence, the net expected payoff of the seller if he invites buyer 2 is
\[
3 \times \frac{1}{35} + 3 \times \frac{18}{35} + 10 \times \frac{16}{35} - 2c = 6.2 - 2c.
\]
Since by assumption, $c < 2$, $3 - c < 6.2 - 2c$, so the seller will invite buyer 2. Note that before inviting any buyer, the expected payoff of the seller is

$$(0.3 + 0.35) \times (1 - c) + 0.35 \times (6.2 - 2c) = 2.82 - 1.35c > 0,$$

where the inequality follows because $c < 2$. Hence, the seller will invite buyer 1 to begin with. We therefore showed that under full information, the optimal search procedure has the following structure:

- Invite buyer 1 first.
- If $v_1 = 1$ or $v_1 = 10$, sell to buyer 1.
- If $v_1 = 3$, invite buyer 2 and then sell to the buyer with the highest valuation between them.

Note that at the optimum, buyer 1 gets the object for sure if $v_1 = 1$ or $v_1 = 10$, and gets it with probability between 0.3 and 0.65 if $v_1 = 3$ (if $v_1 = 3$, the seller eventually sells to buyer 1 if $v_2 = 1$, sells to buyer 2 if $v_2 = 10$, and sells to either buyer if $v_2 = 3$).

### 5.2 The optimal search procedure is not implementable

We now show that there does not exist an incentive compatible mechanism that implements the optimal search procedure. Let $\pi_1(v_1)$ and $t_1(v_1)$, respectively, be the probability that buyer 1 gets the object and the payment he makes, when his valuation is $v_1$. Incentive compatibility for buyer 1 requires (among other things) that

$$3 \times \pi_1(3) - t_1(3) \geq 3 \times \pi_1(1) - t_1(1),$$

$$1 \times \pi_1(1) - t_1(1) \geq 1 \times \pi_1(3) - t_1(3).$$

That is, buyer 1 with valuation $v_1 = 3$ should not pretend that his valuation is $v_1 = 1$ and vice versa. Adding the two inequalities we get that incentive compatibility requires that $\pi_1(3) \geq \pi_1(1)$. However, since at the optimum buyer 1 gets the object for sure if $v_1 = 1$ or $v_1 = 10$, and gets it with probability between 0.3 and 0.65 if $v_1 = 3$, it follows that in order to implement the optimal search procedure, the mechanism must be such that $\pi_1(1) = \pi_1(10) = 1$, and $\pi_1(3) \in [0.3, 0.65]$. This violates the requirement that $\pi_1(3) \geq \pi_1(1)$.

The example indicates a more general result: any non monotonic search procedure cannot be implemented with an incentive compatible mechanism.
To see why, consider a general case where \( v_1 \) can possibly take many values. Then incentive compatibility requires that for any \( v_1 \) and \( v'_1 \neq v_1 \),
\[
\begin{align*}
v_1 \times \pi_1(v_1) - t_1(v_1) & \geq v_1 \times \pi_1(v'_1) - t_1(v'_1), \\
v'_1 \times \pi_1(v'_1) - t_1(v'_1) & \geq v'_1 \times \pi_1(v_1) - t_1(v_1).
\end{align*}
\]
Adding the two inequalities and assuming without a loss of generality \( v_1 > v'_1 \), we get that
\[
(v_1 - v'_1) \times (\pi_1(v_1) - \pi_1(v'_1)) \geq 0.
\]
Since \( v_1 > v'_1 \), it follows that \( \pi_1(v_1) \geq \pi_1(v'_1) \): that is, the higher buyer 1’s valuation is, the higher should be the probability with which buyer 1 gets the object. Hence, it is clear that an incentive compatible mechanism cannot implement a search procedure which is non monotonic like the one in the example above.

The reason why the first-best solution is not always implementable even though the Crémer-McLean (1985, 1988) full-rank condition holds is that we consider a sequential mechanism whereby the seller gets a report from buyer 2 only if he does not sell to buyer 1. Hence, it is not always possible to use buyer 2’s report in order to punish buyer 1 for misreporting his type. In other words, in Crémer and McLean, all buyers know their valuations at the outset so the seller asks all of them to report their valuations and then conditions the payment of each buyer on the reports of other buyers. But since here information acquisition is costly, it may not be optimal for the seller to let all buyers acquire information; as a result, the seller will simply not have reports from all buyers and hence may not be able to induce buyers who do acquire information to make truthful reports.

This begs the question of how does the second-best optimal mechanism looks like when the first-best outcome cannot be implemented?

Tirole suggested that the mechanism would be such that with a small probability the seller moves to buyer 2 even if it is optimal to sell to buyer 1. The advantage of this scheme is that when buyer 1 reports his valuation, he realizes that with a small probability, the seller will obtain a report from buyer 2 and given this report he may have to pay the seller some amount of money. This amount will be larger of course when, given buyer 2’s report, it is likely that buyer 1 misreported his valuation. These punishments may then deter buyer 1 from misreporting his valuation. Thus the mechanism will get us \( \varepsilon \) potentially arbitrarily close to full rent extraction since the probability with which the seller invites buyer 2 to acquire information when it is optimal to sell to buyer 1 could be made small (only the expected punishments matter).
Giving buyers incentives to gather information

Up to now, we have assumed that the mechanism designer could force the buyers to get information. The aim of this section is to study what happens when the seller can only control the buyers’ access to information but cannot actually force them to get it (i.e., the seller can prevent the buyers from getting information but cannot force them to acquire it). We will study this issue in the \textit{i.i.d.} setup, when only one buyer is invited at the time, and show that a modified version of the mechanism used in section 2 provides buyers with the right incentives to acquire information.

As discussed in Crémér, Khalil and Rochet (1998), there are different ways to model pre-contractual gathering of information. Here, we assume as in Crémér and Khalil (1992) that information gathering is \textit{strategic} in the sense that the buyer needs to spend the amount \(c\) in any case if he buys the object (e.g., auditing a firm that has just been acquired), but can do so either before bidding or after winning the object. In Crémér and Khalil (1992), this assumption induces the principal to offer a contract that deters information gathering. Consequently, the agent is uninformed when he accepts the contract, and this enables the principal to inflict ex-post losses on the agent and thereby increase the efficiency of the contract relative to the Baron-Myerson (1982) contract. In the auction setup of the present paper, information gathering before buying the object is not productive from the seller’s viewpoint, but it is nonetheless socially productive as it allows to sell the object to the highest valuation buyer. This generates interesting new issues.

Notice first that in the optimal search mechanism, it is certainly optimal to spend \(c\) before giving the object to a buyer since there is no loss in doing so, and there can be gains. On the other hand, in the context of an auction, if a buyer does not learn his valuation before bidding, then he saves the cost \(c\) in all the cases in which he does not get the object. Indeed, with two buyers, the mechanism of section 2 provides incentives, under certain parameter values, for the last buyer not to acquire information before bidding.

In order to avoid this problem, we modify the mechanism slightly by assuming that in each round, the buyer who just spent the cost \(c\) and learned his type announces it. As in Section 2, let \(k\) be the constant cutoff value in utility, and let \(\hat{v}_i\) be buyer \(i\)’s announcement of his type. If \(\hat{v}_i \geq k\), buyer \(i\) gets the object at a price \(p_i\) which we will compute shortly. If \(\hat{v}_i < k\), the mechanism continues, so buyer \(i\) gets another chance to win the object in the last round if all other buyers have announced valuations that are less than \(\hat{v}_i\); in that case, buyer \(i\) pays a price equal to the highest announced
valuations of the other buyers.

Given $p_i$ that will be computed below, the modified mechanism will be incentive compatible (each buyer will find it optimal to make a truthful announcement), and because it provides all buyers but buyer 1 more information before they learn their own valuation, it encourages the buyers to acquire information.

6.1 Preliminary computations

Since in the modified mechanism buyers (truthfully) announce their valuations, the last buyer, buyer $N$, can win the object by rejecting the seller’s offer, and then bidding in the second price auction above $v^*(N-1)$ which is the highest valuation among all the $N-1$ buyers that were invited before him. Since buyers need to spend the cost $c$ anyway if they get the object, buyer $N$’s expected payoff if he does not gather information is $E[v_i] - c - v^*(N-1)$. If buyer $i$ gathers information, he buys the object only when $v_i > v^*(N-1)$, so his expected payoff is $E[v_i | v_i > v^*(N-1)] - c - v^*(N-1)$. Since $E[v_i | v_i > v^*(N-1)] > E[v_i]$, buyer $N$ is better-off gathering information. Therefore we only need to study the incentives of buyers $i \in \{1, ..., N-1\}$ to gather information.

To this end, suppose that buyer $i$ announces his valuation truthfully. If $v_i \geq k$, buyer $i$ gets the object immediately at a price of $p_i$ so his payoff is $B_Y^i(v_i; p_i) = v_i - p_i$. If $v_i \leq k$, buyer $i$ gets the object only if $v_i$ exceeds the highest current announcement, $v^*(i-1)$, and also exceeds the subsequent announcements of all the $N-i$ remaining buyers. Let

$$v^{N-i} = \max\{v_j \mid j \in \{i+1, \ldots, N\}\},$$

be the highest valuation among all the buyers that are invited after buyer $i$; since the buyers’ valuations are i.i.d., the cumulative distribution of $v^{N-i}$ is given by $F(\cdot)^{N-i}$. Hence, buyer $i$’s discounted expected payoff when $v^*(i-1) < v_i < k$ is

$$B_i^{N}(v_i) = \delta^{N-i} F(k)^{N-i} \left[ \int_0^{v^*(i-1)} (v_i - v^*(i-1)) dF(v^{N-i} \mid v^{N-i} < k) \right.$$  
$$+ \int_{v^*(i-1)}^{v_i} (v_i - v^{N-i}) dF(v^{N-i} \mid v^{N-i} < k) \],$$

(17)

where $F(v^{N-i} \mid v^{N-i} < k) = \frac{F(v^{N-i})^{N-i}}{F(k)^{N-i}}$ is the cumulative distribution function of $v^{N-i}$, conditional on $v^{N-i}$ being less than $k$ (otherwise the mechanism
does reach the second price auction). The first term in the square brackets is the expected payoff of buyer $i$ when he wins the auction and pays a price of $v^*(i - 1)$ (this occurs when $v^*(i - 1) > v^{N-i}$). The second term is the expected payoff of buyer $i$ when he wins the auction and pays a price of $v^{N-i}$ (this occurs when $v^*(i - 1) < v^{N-i}$).

Integrating by parts and simplifying we get

$$B_i^N(v_i) = \delta^{N-i} \int_{v^*(i-1)}^{v_i} F(v)N^{-i}dv,$$

for all $v^*(i-1) < v_i < k$. When $v_i < v^*(i-1) < k$, the payoff of buyer $i$ is 0 since he loses for sure in the second price auction.

The reserve price $p_i$ is set so as to leave buyer $i$ indifferent between accepting the seller’s offer and rejecting it when $v_i = k$. That is, $p_i$ is defined by the equation $B_i^Y(k; p_i) = B_i^N(k)$. Using this equation, it follows that

$$p_i = k - \delta^{N-i} \int_{v^*(i-1)}^{k} F^{N-i}(v)dv.$$

$p_i$ is defined uniquely as $B_i^Y(k; p_i)$ is a decreasing function of $p_i$ and $B_i^Y(k; 0) = k > B_i^N(k) > 0 = B_i^Y(k; k)$. It is clear that given $p_i$, buyer $i$ will make announce his valuation truthfully both when $v_i < k$ and when $v_i > k$ (in the latter case the buyer may actually announce a higher valuation than $v_i$ but since he wins anyway and only pays $p_i$, he may as well make a truthful announcement).

We are now ready to compare the payoff of a buyer when he gathers information and when he does not.

### 6.2 The incentive to gather information

When buyer $i$ does not gather information, his expected valuation is $E[v_i]$ and his net expected payoff from getting the object is $E[v_i] - c$. In what follows we shall assume that $E[v_i] < k$, where using equation (3), $k$ is implicitly defined by

$$\delta \left[ \bar{v} - \int_{k}^{\bar{v}} F(v)dv \right] - k = c.$$

The assumption that $E[v_i] < k$ implies that an uninformed buyer does not accept the seller’s offer and hence may buy the object only at the second price auction held at the last stage (if the mechanism reaches this stage...
at all). Note that when two or more buyers remain uninformed, then at the second price auction they get exactly the same net payoff if they get the object. Therefore they will bid up to the point where their expected payoffs are equal to 0.

Hence, we will now consider the case where only buyer $i$ does not gather information while all other buyers do. The valuation of buyer $i$ in that case is $E[v_i] - c$. When $E[v_i] - c < v^*(N - 1)$, buyer $i$ does not get the object at the second price auction held at the last stage since at least one other buyer ahead of him in the sequence has a higher valuation. Buyer $i$’s payoff then is 0. When $E[v_i] - c > v^*(N - 1)$, buyer $i$ may get the object in the second price auction provided that no other buyer that is invited after him has a higher valuation. Using equation (18), the expected payoff of buyer $i$ in this case is therefore given by

$$V_i^U(v^*(i - 1); k) \equiv \begin{cases} 
B_i^N(E[v_i] - c) = \delta^{N-i} \int_{v^*(i-1)}^E[v_i] c F(v) N^{-i} dv, & \text{if } v^*(i - 1) < E[v_i] - c, \\
0, & \text{if } v^*(i - 1) > E[v_i] - c,
\end{cases}$$

where using equation (20),

$$E[v_i] - c = E[v_i] + k - \delta \left[ \overline{v} - \int_k^\overline{v} F(v) dv \right]$$

$$= \overline{v} - \int_0^{\overline{v}} F(v) dv + k - \delta \left[ \overline{v} - \int_k^\overline{v} F(v) dv \right]$$

$$= (1 - \delta) \left( \overline{v} - \int_k^\overline{v} F(v) dv \right) + k - \int_0^k F(v) dv.$$
his valuation is

\[ V^I_i(v^*(i - 1); k) \equiv \int_{v^*(i-1)}^{k} B^N_i(v_i) dF(v_i) + \int_{k}^{\overline{v}} B^N_i(v_i; p_i) dF(v_i) - c \]

\[ = \delta^{N-i} \int_{v^*(i-1)}^{k} \left( \int_{v^*(i-1)}^{v_i} F(v)N-i dF(v) \right) dF(v_i) + \int_{k}^{v_i} (v_i - p_i) dF(v_i) - c, \]

where the superscript \( I \) stands for "Informed." After integration by parts and substituting for \( p_i \) from equation (19), the expected payoff of buyer \( i \) becomes

\[ V^I_i(v^*(i - 1); k) = \delta^{N-i} \left[ \left( \int_{v^*(i-1)}^{k} F(v)N-i dF(v) \right) F(k) - \int_{v^*(i-1)}^{k} F(v)N-i+1 dF(v) \right] \]

\[ + (\overline{v} - p_i) - (k - p_i) F(k) - \int_{k}^{\overline{v}} F(v) dF(v) - c \]

\[ = \delta^{N-i} \left[ \left( \int_{v^*(i-1)}^{k} F(v)N-i dF(v) \right) F(k) - \int_{v^*(i-1)}^{k} F(v)N-i+1 dF(v) \right] \]

\[ + \left( \overline{v} - k + \delta^{N-i} \int_{v^*(i-1)}^{k} F(v)N-i dF(v) \right) \]

\[ - \left( \delta^{N-i} \int_{v^*(i-1)}^{k} F^N-i(v) dF(v) \right) F(k) - \int_{k}^{\overline{v}} F(v) dF(v) - c \]

\[ = (\overline{v} - k) - \int_{k}^{\overline{v}} F(v) dF(v) + \]

\[ \delta^{N-i} \left[ \int_{v^*(i-1)}^{k} F(v)N-i dF(v) - \int_{v^*(i-1)}^{k} F(v)N-i+1 dF(v) \right] - c. \]

Finally, substituting for \( c \) from equation (20) and simplifying, we get

\[ V^I_i(v^*(i - 1); k) = (1 - \delta) \left( \overline{v} - \int_{k}^{\overline{v}} F(v) dF(v) \right) + \delta^{N-i} \int_{v^*(i-1)}^{k} F(v)N-i (1 - F(v)) dF(v). \]

To show that buyer \( i \in \{1, ..., N - 1\} \) will choose to gather information, we need to prove the following claim.

**Claim 1** \( V^I_i(v^*(i - 1); k) > V^U_i(v^*(i - 1); k) \) for all \( v^*(i - 1) \leq E[v_i] - c \), all \( k \), and all \( i \in \{1, ..., N - 1\} \).
First, we establish that $V_i^U(0; k) < V_i^I(0; k)$ for all $k$.\footnote{Note that when $v^*(i - 1) > k$, the mechanism does not even reach buyer $i$ as one of the $N - i$ buyers ahead of him in the sequence buys the object. Hence, we shall assume implicitly that $v^*(i - 1) < k$.} To this end, notice that evaluated at $v^*(N - 1) = k = 0$,

$$V_i^U(0; 0) = \delta^{N-i} \int_0^{(1-\delta)(\bar{v}-\int_0^\bar{v} F(v)dv)} F(v)^{N-i} dv,$$

and

$$V_i^I(0; 0) = (1-\delta) \left( \bar{v} - \int_0^\bar{v} F(v)dv \right).$$

Using these expressions, and noting that $\delta$ and $F(v)$ are less than 1, it follows that

$$V_i^U(0; 0) = \delta^{N-i} \int_0^{V_i^I(0;0)} F(v)^{N-i} dv < \delta^{N-i} \int_0^{V_i^I(0;0)} dv = \delta^{N-i} \delta^N = V_i^I(0; 0).$$

Furthermore,

$$\frac{\partial V_i^U(0; k)}{\partial k} = \frac{\partial}{\partial k} \left[ \delta^{N-i} \int_0^{(1-\delta)(\bar{v}-\int_k^\bar{v} F(v)dv)+k-\int_0^k F(v)dv} F(v)^{N-i} dv \right]$$

$$= \delta^{N-i} F \left( 1-\delta \left( \bar{v} - \int_k^\bar{v} F(v)dv \right) + k - \int_0^k F(v)dv \right)^{N-i} \times (1-\delta F(k))$$

$$< \delta^{N-i} F(k)^{N-i}(1-\delta F(k)),$$

where the inequality follows because by definition, $(1-\delta) \left( \bar{v} - \int_k^\bar{v} F(v)dv \right) + k - \int_0^k F(v)dv = E[v_i] - c$ and since by assumption, $E[v_i] - c < k$, and

$$\frac{\partial V_i^I(0; k)}{\partial k} = (1-\delta) F(k) + \delta^{N-i} F(k)^{N-i} (1-F(k))$$

$$> \delta^{N-i} F(k)^{N-i} ((1-\delta) F(k) + 1 - F(k))$$

$$= \delta^{N-i} F(k)^{N-i}(1-\delta F(k)),$$
where the inequality follows because $\delta$ and $F(k)$ are less than 1. Hence, 
$\frac{\partial V_i^I(0; k)}{\partial k} > \delta^{N-i} F(k)^{N-i} (1 - \delta F(k)) > \frac{\partial V_i^U(0; k)}{\partial k}$, for all $k$. Together with the fact that $V_i^U(0; 0) < V_i^I(0; 0)$, this proves that $V_i^U(0; k) < V_i^I(0; k)$ for all $k$.

Second, note that for all $v^*(i-1) < E[v_i] - c$, 
\[
\frac{\partial V_i^U(v^*(i-1); k)}{\partial v^*(i-1)} = -\delta^{N-i} F(v^*(i-1))^{N-i},
\]
and
\[
\frac{\partial V_i^I(v^*(i-1); k)}{\partial v^*(i-1)} = -\delta^{N-i} F(v^*(i-1))(1 - F(v^*(i-1))) (22)
\]
Both derivatives are negative, but since $|\frac{\partial V_i^U(v^*(i-1); k)}{\partial v^*(i-1)}| \geq |\frac{\partial V_i^I(v^*(i-1); k)}{\partial v^*(i-1)}|$ for all $v^*(i-1) < E[v_i] - c$ and all $k$, and since $V_i^I(0; k) > V_i^U(0; k)$ for all $k$, it follows that $V_i^I(v^*(i-1); k) > V_i^U(v^*(i-1); k)$ for all $v^*(i-1) < E[v_i] - c$, all $k$, and all $i \in \{1, ..., N-1\}$. Q.E.D

The claim proves that if all other buyers gather information, so will buyer $i \in \{1, ..., N-1\}$. Since we already showed earlier that buyer $N$ also gathers information, this establishes that under the proposed mechanism, all buyers will choose to gather information.

There are now two open questions: first, what is the intuition for the result that it is always better to acquire information in advance rather than wait. Second, the question is whether this result generalizes to the general search problem environment or whether it is only true in the i.i.d. case?

7 References


