A Primer on the Limit Pricing of Goods with Network Effects

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Abstract

This paper establishes and describes limit pricing Markov-perfect equilibria in a two-player entry deterrence game where the market demand for the good is characterized by network effects. The range of prices consistent with limit pricing equilibria and the range of fixed costs that give rise to limit pricing equilibrium are found to increase with the strength of the network effect.

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†Super-preliminary. Please do not quote. This paper is in no way meant to convey any opinions about any particular firm or market. The views expressed here are of the author and not necessarily Analysis Group.

1 Introduction

Limit pricing equilibria are often advanced to explain the persistence of a firm's dominance in markets where there are no obvious barriers to entry or technical advantages enjoyed by the firm. In a limit pricing equilibria, a dominant firm prices below its short-run profit maximizing optimum in order to deter entry by potential competitors or limit in some way existing competitors in future periods. In order for such equilibria to exist, there must be a dynamic link between pricing behavior in the current period and future market conditions[1]. When such a mechanism exists, a dominant firm may choose to deter potential future competition by incurring the opportunity cost of sub-optimal pricing in order to alter conditions in future periods to their advantage.

Many examples of such mechanisms that link current with future markets have been explored in the literature. some prominent examples include the endogenous capacity of a "fringe" of small competitors (Gaskins [5]) , asymmetric information on costs (Milgrom and Roberts [7]), advertising (Bagwell and Ramey [2]) and most recently network effects in demand (Fudenberg and Tirole [4]). While historically the link between current and future periods has often been on the supply side, in Fudenberg and Tirole [4] and this paper, we focus on an important demand side link: network effects in demand.

In this note, network effects means simply that the utility derived (and hence the amount demanded at any price) by a consumer for a good depends positively on the number of other people who own the good. Motivating examples are easily found in markets for goods where standards are important; examples include VCR's, Compact disks, DVD's and computer software (for a discussion see Leibowitz and Margolis [6]).

The approach outlined here follows in the spirit of though not exactly the approach of Fudenberg and Tirole [4]. In their paper, Markov-perfect equilibria are established for an infinitely repeated game with overlapping generations of consumers in which an incumbent prices a network good. Potential competitors with superior technology and stochastic costs appear each period and may displace an incumbent. Limit pricing in their model occurs when the incumbent's strategy in equilibrium is to price below the short run optimum to sell to low value types in order to change the likelihood of being displaced in future periods. In this paper, an incumbent faces a single potential entrant with known costs and a similar product in a two-period game.

The next section describes the model and derives the equilibrium. The third section examines some comparative statics of the model equilibria, while the fourth section concludes. An appendix contains some of the details for deriving the equilibrium.

2 A Simple Model

Two firms may compete in a market where demand for the good is characterized by network effects. That is, the value of the good to a buyer increases with the number of other people

[1]Without some formal link across time, pricing behavior in the current period places no constraints on future pricing behavior and the incumbent can offer no credible deterrence to entry. See Friedman [3].

who own it[2]. In all other respects we will assume the goods produced by the two firms are interchangeable in terms of their utility to the consumer. In the first period, the dominant firm (Firm 1) is alone in the market. Demand is given by:

$$q = e^{-\lambda p} \tag{1}$$

where p is the price charged by the firm, q is quantity sold and $\lambda > 0$ is a known constant. In the second period a second firm (Firm 2) may enter the market. In the second period we will subscript prices and quantities. Demand for each firm in the second period is determined by differentiated product Bertrand competition, specifically:

$$q_1 = ze^{-\lambda p_1} \quad \text{and} \quad q_2 = (1-z)e^{-\lambda p_2} \tag{2}$$

where $\{q_1, p_1\}$ and $\{q_2, p_2\}$ are the quantities demanded and prices charged from Firm 1 and Firm 2 respectively. z denotes the market share of customers for each of the two goods and is given by a logit equation:

$$z = \frac{e^{q-\theta p_1}}{e^{q-\theta p_1} + e^{-\theta p_2}} \tag{3}$$

This equation captures network effects in our model. In terms of attracting market share for customers, Firm 1 enjoys an advantage over Firm 2 that depends on sales in the first period [3]. Hence an investment in creating a large network of buyers in the first period (at the opportunity cost of charging less than the monopoly price) yields dividends in the second period in terms of larger market share at any price *ceteris paribus* when entry occurs.

In order to simplify the analysis, assume there is a fixed cost F to production and a constant marginal cost of $c = 0$. Costs are symmetric for the two firms and fixed costs increase at the discount rate r per period.

Notice that this model lacks any real treatment of network effects within the first period. If the first period is taken to be a short interval of time for a new product, this may not be far off the mark as sales are literally to the first adopters of a new technology. However given a long enough interval sales at the beginning of the interval should affect sales at the end of the interval when network effects are present. Since our interest here is on pricing to deter entry, I do not account for this but it could easily be incorporated into the model.

A second limitation of the model is that it does not model any inter-temporal substitution of demand. While stimulating demand through network effects is our focus here, we could also address market "saturation" in the short run whereby the incumbent lowers short-run future demand by discounting in the current period. While we maintain independence between the size of the two markets in our model, allowing some interdependence would probably be more realistic but at the cost of complicating the analysis.

[2] This notion is not new and dates back to at least the work of Artle and Averous [1], Rohlfs [8], Squire [9] and von Rabenau and Stahl [10]

[3] Another approach would be to have demand depend on the number of past and contemporaneous sales of the good. This would put a substantial burden on the information the consumer is assumed to have in a particular period. Moreover there would still be an advantage enjoyed by the incumbent based on sales in the first period.

Strategies for Firm 1 are its prices in the first and second period. We rule out uninteresting cases where demand is insufficient to entice Firm 1 to participate in the market in the first period ($F < \frac{1}{\lambda}e^{-1}$). Firm 2's strategies are whether or not to enter in the second period and if entry is made, the price to charge.

The solution concept we will consider is Markov-perfect equilibria. Hence our focus will be on closed loop equilibria. In a Markov-perfect equilibrium, strategies may depend on history only through a state variable. In our model this will be the observed sales of the incumbent in the first period.

2.1 Equilibrium

We solve for our equilibrium by backwards induction. Given any base $q = B$ for the incumbent generated by its price in the first period, we may solve for equilibrium prices in the second period by jointly solving the first order conditions for profit maximization for the two firms in the second period. It is shown in the appendix that conditional on base of sales B, profit maximizing prices for the two firms are given by

$$p_1^* = \frac{1}{\theta(1-z^*)+\lambda} \qquad p_2^* = \frac{1}{\theta z^* + \lambda} \tag{4}$$

where $z*$ solves

$$log(z^*) = B - \frac{\theta}{\theta(1-z^*)+\lambda} - log(e^{B-\frac{\theta}{\theta(1-z^*)+\lambda}} - e^{\frac{\theta}{\theta z^*+\lambda}}) \tag{5}$$

This constitutes the unique rationalizable (and hence Nash) equilibrium for the second period conditional on Firm 2's entry (see Appendix). Hence solving this problem for all value of B generates a second period revenue function for Firm 1 as a function of its price in the first period. Let's denote this revenue function ψ. This describes payoffs to Firm 1 from all continuation sub-games in which Firm 2 enters the market. Hence if Firm 1 anticipates entry, it solves:

$$p^* = argmax_{p\in[0,\infty)}\{pq + \frac{1}{1+r}\psi(p)\}$$

where r is Firm 1's discount rate. The first order condition is

$$(1-\lambda p)q + \frac{1}{1+r}\psi'(p) = 0 \tag{6}$$

The solution to (6) will determine Firm 1's best response to an entering strategy for Firm 2. Equation (6) differs from the simple monopoly case by the addition of the derivative of the second period revenue function. Notice that since $\psi'(p) < 0$ the optimal strategy is for the incumbent to invest in a network and price below the myopic monopoly price of $1/\lambda$ when it anticipates entry.

From Firm 2's point of view, the entry decision turns on whether the revenue it receives given B exceeds its fixed costs F. That is conditional on observing Firm 1's first period sales B, entry will occur if:

$$\frac{1}{\theta z^*+\lambda}(1-z^*)e^{\frac{-\lambda}{\theta z^*+\lambda}} \geq F \tag{7}$$

Table 1: Second Period Prices, Incumbent's Market Share and Range of Limit Pricing Equilibria

θ	r	p_1	p_2	Share	Range
0	0.1	1	1	59.9%	0.022
0.1	0.1	0.961	0.944	59.4%	0.025
1	0.1	0.704	0.633	56.3%	0.023
1	0.2	0.704	0.633	56.2%	0.020
10	0.1	0.178	0.156	53.4%	0.010
1000	0.1	0.002	0.002	53.1%	0.0001

where again z^* is given by (5). Call the level of first period sales B which satisfies (7) as an equality plus one unit as B^*. Within this framework, limit pricing equilibria obtains when the revenue to Firm 1, $(1/\lambda)\{\frac{1}{(1+r)}e^{-1} - B^* log(B^*)\}$ under a monopoly constrained to deter entry exceeds the duopoly revenue implied by the solution to (6). Hence depending on fixed costs F three types of equilibria may obtain. When F is close to zero, Firm 2 can not be deterred and enters the market in period two, Firm 1 anticipates this and the equilibrium is given by (4), (5) and (6). When F is large enough the margin for Firm 2 is small enough that Firm 1 can foreclose it by a limit price in the first period. In this case Firm 1 chooses a low price in period one, Firm 2 stays out in period two and Firm 1 enjoys monopoly rents in the second period. Finally if F is large, the market is a natural monopoly and Firm 2 does not enter even when Firm 1 chooses the short-run monopoly price. Figure 1 shows the equilibrium price in the first period (and hence the regime) as a function of F for three different values of θ and two values of the discount rate. Here $\lambda = 1$ so that the unconstrained short-run monopoly price is unity. Since θ is the coefficient paired with price, it measures the ferocity of price competition and is inversely related to the strength of the network effect at the *margin*.

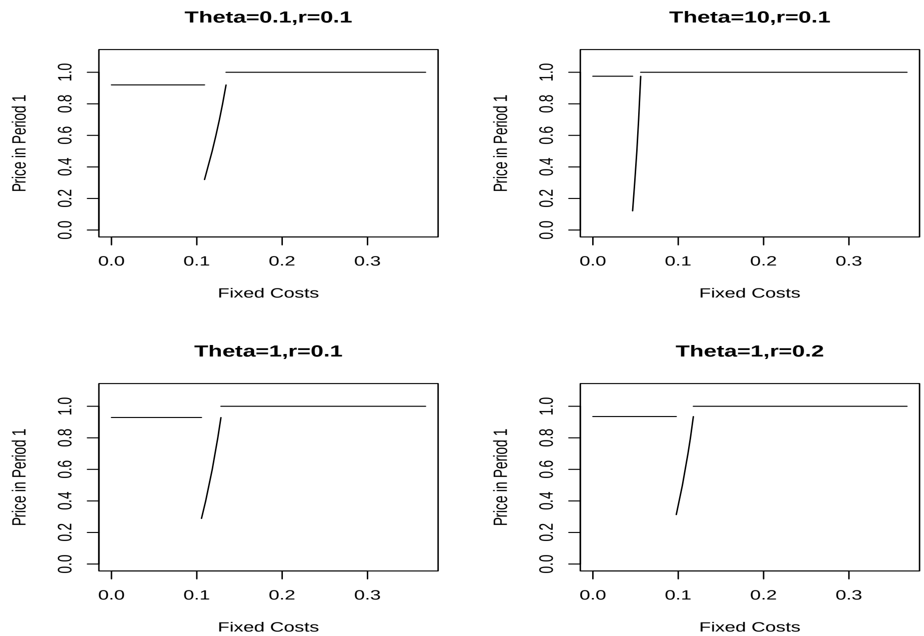
In each of the four scenarios, the upward sloping portion of the graph corresponds to the limit pricing equilibria. Discontinuities occur where there is regime shift from duopoly to constrained monopoly to unconstrained monopoly (respectively as fixed costs increase). As we increase the strength of price competition, the range of prices sustainable as a limit price equilibrium increases while the range of fixed costs which are associated with limit pricing decreases. Costs are measured in first period dollars.

3 Some Comparative Statics

The parameter θ is the strength of price competition, and as it approaches ∞ the incumbent's advantage can be nullified by an arbitrarily small discount in second period price from the entrant. As θ approaches zero, market share is largely determined by the fixed network effect. Note that there is no coefficient multiplying period 1 sales in (3). Hence there is a fixed network effect competing with varying levels of price competition.

Table 1 shows the accompanying second period prices, market share and the range of

Figure 1: Equilibrium Price in the First Period



costs associated with limit pricing equilibrium. Two extreme cases are also reported in Table 1. The case where $\theta = 0$ gives the case where first period sales alone determine the share of customers, while the case $\theta = 1000$ gives the case where pricing is decisive at the margin for Firm 1. In the former case, the products are so differentiated that each firm has a monopoly franchise and prices accordingly. In the latter case, price competition is fierce and drives second period prices towards zero.

Examining Figure 1 and Table 1, we see that for a good with moderate network effects in demand (when price competition is non-existent, the incumbent obtains only about 60 percent of the market) when the products are largely differentiated, the equilibria follow as in the first graph in Figure 1. As price competition increases, the lower bound of feasible limit prices drops to zero as the range of fixed costs over which they occur also drop to zero. However as a proportion of the revenue available to an entrant the range of costs under which limit pricing equilibria obtain is remarkable stable at between 16 and 19 percent– see Table 2.

The discount rate governs the value of the second period monopoly under a limit pricing equilibrium. For this reason, we would expect the size of the region of costs which generate limit-pricing equilibria to drop as r increases but this is not the case comparing the two sets of equilibria in Figure 1 with disparate discount rates. The reason for this is that since the revenue to Firm 2 in the second period (which equals cost under limit pricing) is governed by the same margin, the overall effect of the increase in r is washed out. Insofar as the risk associated with future revenue for Firm 2 differs from the risk of future costs, introducing

Table 2: Range of Limit Pricing and Marginal Effect

θ	r	Range as Percentage	Slope
0	0.1	16.0%	24.0
0.1	0.1	18.8%	24.1
1	0.1	17.0%	29.2
1	0.2	17.0%	31.6
10	0.1	16.6%	93.7
1000	0.1	17.7%	6425.4

a separate discount rate for fixed costs would alter this result.

We can also examine equilibria when we fix the level of price competition θ and vary the strength of the network effect by introducing a new parameter α into equation (3):

$$z = \frac{e^{\alpha q - \theta p_1}}{e^{\alpha q - \theta p_1} + e^{-\theta p_2}} \tag{8}$$

Holding the price competition parameter constant at $\theta = 1$, we can vary α to see the effect of increasing the strength of the network effect. The results are given in Figure 2.

We see that the proportion of costs relative to the entrant's duopoly revenue increases with the network effect– from 1.9 to 43 percent as α increases from 0.1 to 10.

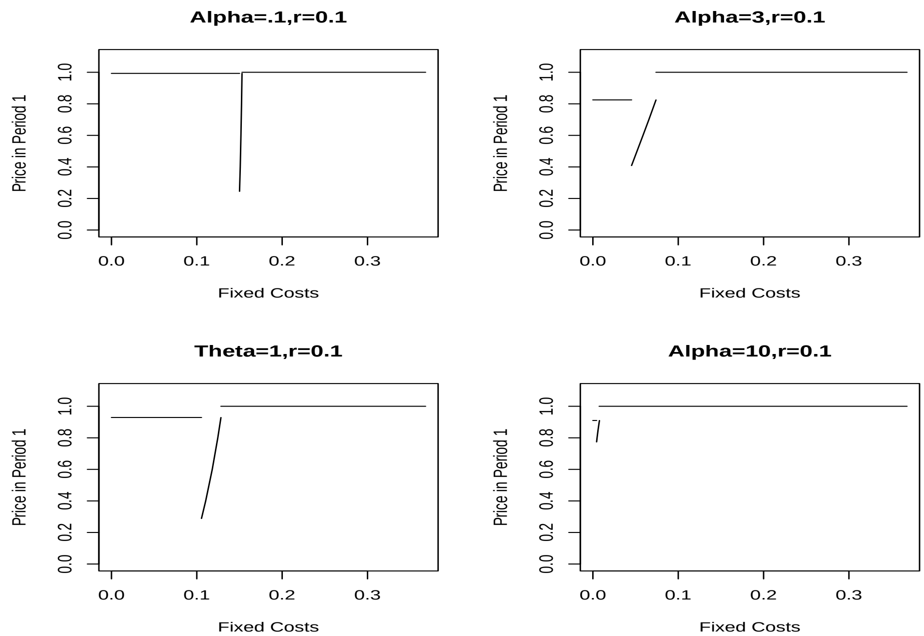
4 Discussion

This note introduces a simple two-period model of entry deterrence based on only the network effects present in the demand for the good. The Markov-perfect equilibria vary according to fixed cost of production and fall into three categories: unconstrained monopoly in both periods, constrained (limit pricing) monopoly in the first period, and constrained monopoly followed by competition in the second period.

The range of prices consistent with limit pricing equilibria and the range of fixed costs that give rise to limit pricing equilibrium are found to increase with the strength of the network effect. Since the network effect measures a return on the investment in future demand this is as expected.

An interesting issue that has been sidestepped in this note is a careful analysis on the role of consumers. For example, we do not treat the issues of beliefs about the future and coordination among consumers. Here consumers of the good behave myopically: they observe sales and choose a brand. Consumers are probably more forward looking and may make forecasts as to the viability of competing products into the future. Consumers may also try to coordinate in choosing among equilibria.

Figure 2: Equilibrium Price in the First Period



Appendix

Derivation of Prices in the Second Period

Since $z = \eta_1/(\eta_1 + \eta_2)$ where $\eta_1 = e^{B-\theta p_1}$ and $\eta_2 = e^{-\theta p_2}$, when entry occurs, we have

$$\frac{\partial z}{\partial p_1} = \frac{(\eta_1 + \eta_2)(-\theta\eta_1) - \eta_1(-\theta\eta_1)}{(\eta_1 + \eta_2)^2} = -\theta z(1-z)$$

Similarly $\frac{\partial z}{\partial p_2} = \theta z(1-z)$. This gives

$$\frac{\partial y_1}{\partial p_1} = (\frac{\partial z}{\partial p_1})e^{-\lambda p_1} - \lambda z e^{-\lambda p_1} = [-\theta z(1-z) - \lambda z]e^{-\lambda p_1}$$

and similarly $\frac{\partial y_2}{\partial p_2} = [-\theta z(1-z) - \lambda(1-z)]e^{-\lambda p_2}$. Setting the first order conditions for revenue maximization equal to zero holding the other firm's prices constant gives prices:

$$p_1^* = \frac{1}{\theta(1-z) + \lambda} \qquad p_2^* = \frac{1}{\theta z + \lambda}$$

assuming Firm 2 chooses to enter the market. Substituting these values back into the definition gives a nonlinear equation in z which we can solve numerically. That is, we find the root of the following equation:

$$log(z^*) = B - \frac{\theta}{\theta(1-z^*) + \lambda} - log(e^{B - \frac{\theta}{\theta(1-z^*)+\lambda}} - e^{\frac{\theta}{\theta z^* + \lambda}})$$

Uniqueness of Sub-game Equilibrium

The best response correspondence for any pure strategy for Firms 1 and 2 are given by the equations for p_1^* and p_2^* above. The dependence of the best response on the price chosen by the other firm flows through the z, and since $\frac{\partial z}{\partial p_2} < 0$ then $\frac{\partial p_1^*}{\partial p_2} > 0$ Hence the best response correspondence $p_1^*(p_2)$ is monotonically increasing from $\frac{1}{\theta(1-\bar{z})+\lambda}$ to $\frac{1}{\lambda}$ where $\bar{z} = \frac{e^{B-\lambda p_1^*}}{e^{B-\lambda p_1^*}+1}$. Similarly for $p_2^*(p_1)$. Hence the unique rationalizable strategy pair satisfies the three equations for p_1^*, p_2^* and z^* above.

References

[1] Artle R. and Averous C. (1973): "The Telephone System as a Public Good: Static and Dynamic Aspects" *Bell Journal of Economics and Management Science*, 4(1), 89-100.

[2] Bagwell K. and Ramey G. (1988): "Advertising and Limit Pricing" *RAND Journal of Economics*, 29(1), 55-71.

[3] Friedman J. (1979):"On Entry Preventing Bahavior" in *Applied Game Theory* ed. by Brams, Schotter and Schwodiauer, Worzberg Vienna 236-253.

[4] Fudenberg D. and J. Tirole (2000): "Pricing a Network Good to Deter Entry" *Journal of Industrial Economics*, 48(4), 373-390.

[5] Gaskins D. (1970): "Dynamic Limit Pricing: Optimal Pricing under Threat of Entry" *Journal of Economic Theory* 3, 306-322.

[6] Leibowitz S. and S. Margolis (1999): *Winners, Losers and Microsoft*, The Independent Institute, Oakland, CA.

[7] Milgrom P. and J. Roberts (1982): "Limit Pricing and Entry under Incomplete Information: An Equilibrium Analysis" *Econometrica*, 50(2), 443-459.

[8] Rohlfs J. (1973): "A Theory of Interdependent Demand for a Communications Service" *Bell Journal of Economics and Management Science*, 5(1), 16-37.

[9] Squire L. (1973): "Some Aspects of Optimal Pricing of Telecommunications" *Bell Journal of Economics and Management Science*, 4(2), 515-525.

[10] von Rabenau B. and Stahl K. (1974): "Dynamic Aspects of Public Goods: A Further Analysis of the Telephone System" *Bell Journal of Economics and Management Science*, 5(2), 651-669.